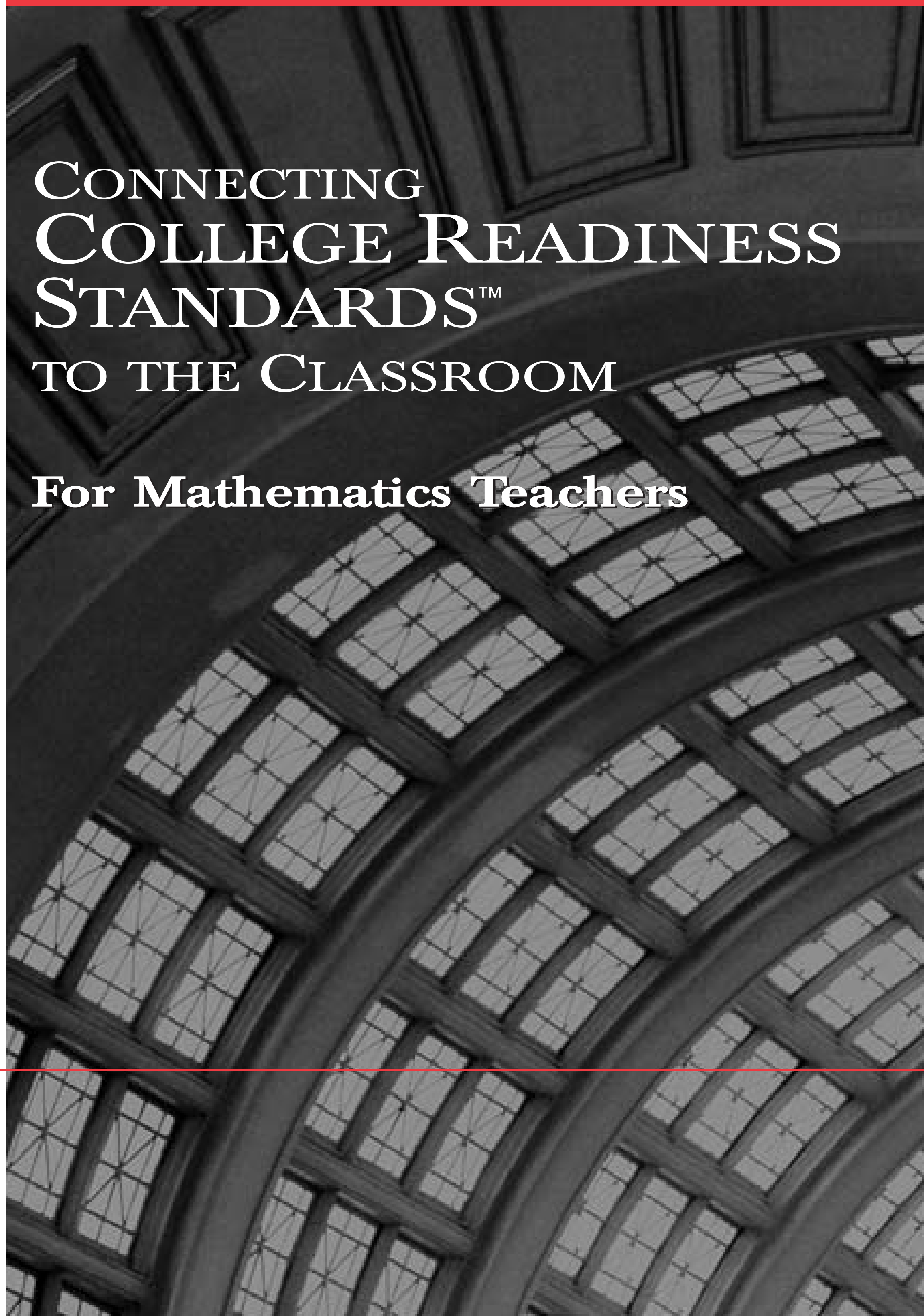
The ACT®

Connecting College Readiness Standards™ to the Classroom

For Mathematics Teachers



ACT®

ACT endorses the *Code of Fair Testing Practices in Education* and the *Code of Professional Responsibilities in Educational Measurement*, guides to the conduct of those involved in educational testing. ACT is committed to ensuring that each of its testing programs upholds the guidelines in each *Code*.

A copy of each *Code* may be obtained free of charge from ACT Customer Services (68), P.O. Box 1008, Iowa City, IA 52243-1008, 319/337-1429.

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INTRODUCTION

ACT has developed this guide to help classroom teachers, curriculum coordinators, and counselors interpret the College Readiness Standards™ report for ACT Mathematics. The guide includes:

- A description of the **College Readiness Standards** for the ACT®
- A description of the **ACT Mathematics Test**
- A set of sample **test questions**
- A description of the **Assessment-Instruction Link**
- A set of classroom **instructional activities**

The College Readiness Standards for the ACT are statements that describe what students who score in the six score ranges 13–15, 16–19, 20–23, 24–27, 28–32, and 33–36 on the multiple-choice tests and in the five score ranges 3–4, 5–6, 7–8, 9–10, and 11–12 on the Writing Test are *likely* to know and to be able to do. The statements are generalizations based on the performance of many students. College Readiness Standards have not been developed for students whose scores fall in the 1–12 range for the multiple-choice tests and at score point 2 for the Writing Test because these students, as a group, do not demonstrate skills similar to each other consistently enough to permit useful generalizations.

The College Readiness Standards for the ACT are accompanied by ideas for progress that help teachers identify ways of enhancing student learning based on the scores students receive.

The College Readiness Standards Information Services provide six aggregate reports for the ACT. Five of these reports are content specific: each presents the scores of your most recent graduates in one of the five content areas the ACT test measures—English, Mathematics, Reading, Science, and Writing. These five content-specific reports present the ACT results using ACT's College Readiness Standards. The sixth report, the Summary Profile, summarizes the scores, across all five content areas, of your most recent graduating class who tested as tenth, eleventh, or twelfth graders. All six reports provide data that compare the performance of your school's most recent graduating class with the performance of two norm groups: national and state. The data in the reports reflect the characteristics of those students who either took the ACT on a national test date or as part of a state testing initiative and who reported that they plan to graduate from high school during the most recent academic year.

The ACT is a curriculum-based assessment program developed by ACT to help students prepare for the transition to postsecondary education while providing a measure of high school outcomes for college-bound students. As part of ACT's Educational Planning and Assessment System (EPAS™), the ACT is complemented by EXPLORE®, ACT's eighth- and ninth-grade program, and by PLAN®, for tenth graders. We hope this guide helps you assist your students as they plan and pursue their future studies.

"The role of standardized testing is to let parents, students, and institutions know what students are ready to learn next."

— Ralph Tyler, October 1991
Chairman Emeritus of
ACT's Board of Trustees

The College Readiness Standards Report for ACT Mathematics

The College Readiness Standards report for ACT Mathematics allows you to compare the performance of students in your school with the performance of students at the national and state levels. The report provides summary information you can use to map the development of your students' knowledge and skills in mathematics. Used along with your own classroom observations and with other resources, the test results can help you to analyze your students' progress in mathematics and to identify areas of strength and areas that need more attention. You can then use the Standards as one source of information in the instructional planning process.

A sample report appears on the next page. An explanation of its features is provided below.

A This section briefly explains the uses of the report to help you interpret the test results.

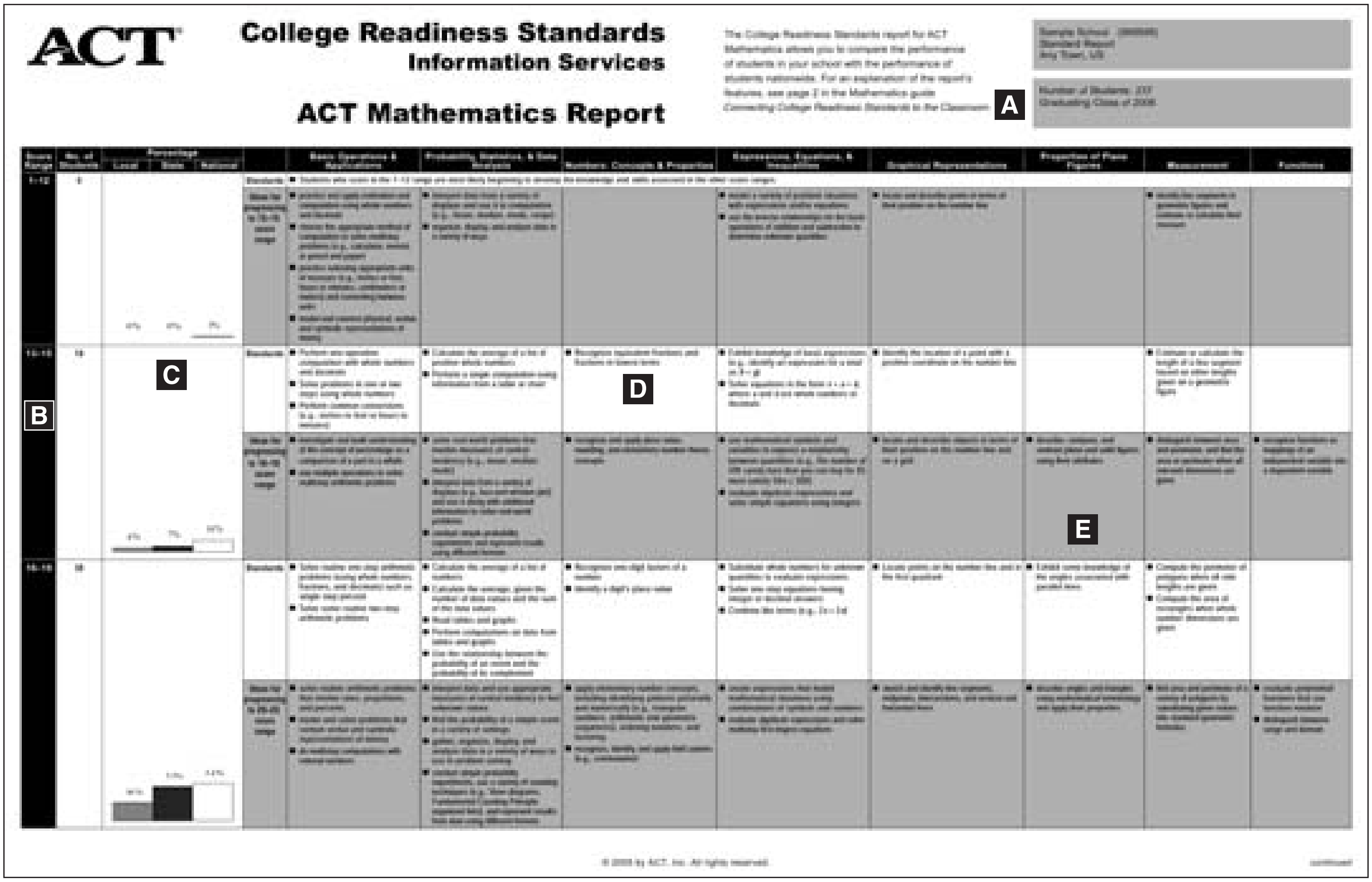
B These are the seven score ranges reported for the College Readiness Standards for the ACT. To determine the number of score ranges and the width of each score range, ACT staff reviewed normative data, college admission criteria, and information obtained through ACT's Course Placement Service. For a more detailed explanation of the way the score ranges were determined, see page 5.

C This section compares the percent of graduating seniors who tested as tenth, eleventh, or twelfth graders and who scored in a particular score range at an individual school (Local) with the percent of all graduating students in the national and state norm groups who scored in the same range. The percent of students at the local school and for the national and state groups are based on the performance of students who either took the ACT on a national test date or as part of a state testing initiative and who reported that they plan to graduate from high school during the most recent academic year. The number of local school students who scored in each of the seven score ranges is provided in the column to the left of each bar graph; the total number of graduating students tested locally is provided at the top of the report.

D The College Readiness Standards were developed by identifying the knowledge and skills students need in order to respond successfully to questions on the ACT Mathematics Test. As you review the report for ACT Mathematics, you will note that the Standards are cumulative, which means that if students score, for example, in the 20–23 score range, they are likely to be able to demonstrate most or all of the knowledge and skills in the 13–15, 16–19, and 20–23 score ranges. Students may be able to demonstrate some of the skills in the next score range, 24–27, but not consistently enough as a group to reach that score range. A description of the way the College Readiness Standards were developed can be found on pages 5–6.

E The "ideas for progress" are statements that provide suggestions for learning experiences that students might benefit from. These ideas for progress are arranged by score range and strand. Although many of the ideas cross more than one strand, a primary strand has been identified for each in order to facilitate their use in the classroom. Ideas for progress are not provided for the 33–36 score range, the highest score range for the ACT. Students who score in this range on the ACT Mathematics Test have demonstrated proficiency in all or almost all of the skills measured by the test.

F Page 2 of the report profiles the test results, College Readiness Standards, and ideas for progress for score ranges 20–23, 24–27, 28–32, and 33–36.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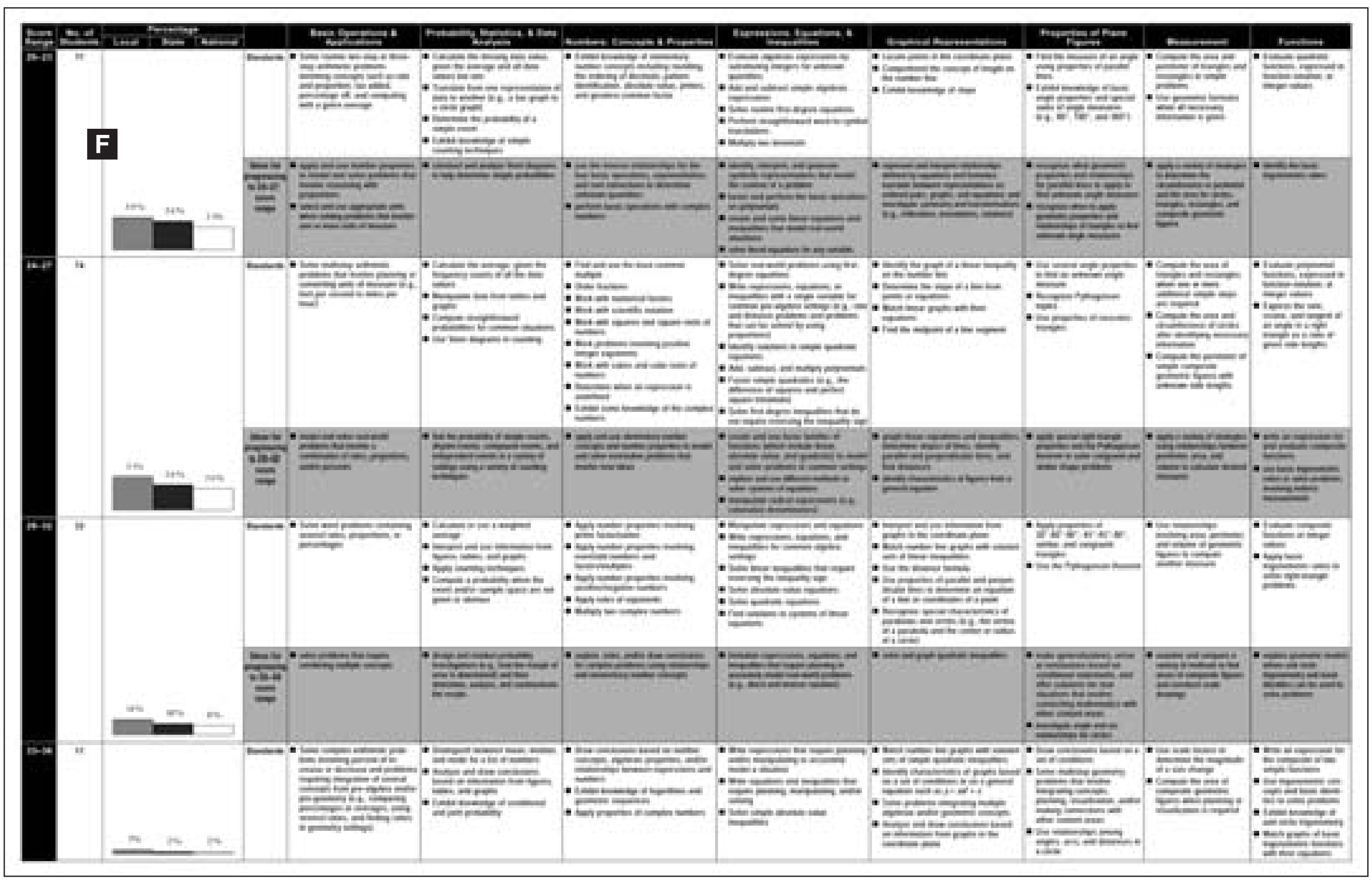
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Description of the College Readiness Standards

What Are the College Readiness Standards?

The College Readiness Standards communicate educational expectations. Each Standard describes what students who score in the designated range are *likely* to be able to do with what they know. Students can typically demonstrate the skills and knowledge within the score ranges preceding the range in which they scored, so the College Readiness Standards are cumulative.

In helping students make the transition from high school to postsecondary education or to the world of work, teachers, counselors, and parents can use the College Readiness Standards for the ACT to interpret students' scores and to understand which skills students need to develop to be better prepared for the future.

How Were the Score Ranges Determined?

To determine the number of score ranges and the width of each score range for the ACT, ACT staff reviewed ACT normative data and considered the relationship among EXPLORE, PLAN, and the ACT.

In reviewing the ACT normative data, ACT staff analyzed the distribution of student scores across the score scale, 1–36. Because the ACT is used for college admission and course-placement decisions, differing admission criteria (e.g., open, liberal, traditional, selective, and highly selective) and the course-placement research that ACT has conducted over the last forty years were also reviewed. ACT's Course Placement Service provides colleges and universities with cutoff scores that are used to place students into appropriate entry-level courses in college; and these cutoff scores were used to help define the score ranges.

After analyzing all the data and reviewing different possible score ranges, ACT staff concluded that using the seven score ranges 1–12, 13–15, 16–19, 20–23, 24–27, 28–32, and 33–36 would best distinguish students' levels of achievement so as to assist teachers, administrators, and others in relating ACT test scores to students' skills and understandings.

How Were the College Readiness Standards Developed?

After a review of normative data, college admission criteria, and information obtained through ACT's Course Placement Service, content experts wrote the College Readiness Standards based on their analysis of the skills and knowledge students need in order to successfully respond to the test questions in each score range. Experts analyzed numerous test questions that had been answered correctly by 80% or more of the examinees within each score range. The 80% criterion was chosen because it offers those who use the College

"The examination should describe the student in meaningful terms–meaningful to the student, the parent, and the elementary and high school teacher–meaningful in the sense that the profile scores correspond to recognizable school activities, and directly suggest appropriate distributions of emphasis in learning and teaching."

— E. F. Lindquist, February 1958
Cofounder of ACT

Readiness Standards a high degree of confidence that students scoring in a given score range will most *likely* be able to demonstrate the skills and knowledge described in that range.

As a content validity check, ACT invited nationally recognized scholars from high school and university Mathematics and Education departments to review the College Readiness Standards for the ACT Mathematics Test. These teachers and researchers provided ACT with independent, authoritative reviews of the ways the College Readiness Standards reflect the skills and knowledge students need to successfully respond to the questions on the ACT Mathematics Test.

Because the ACT is curriculum based, ACT and independent consultants conduct a review every three to four years to ensure that the knowledge and skills described in the Standards and outlined in the test specifications continue to reflect those being taught in classrooms nationwide.

How Should the College Readiness Standards Be Interpreted and Used?

The College Readiness Standards reflect the progression and complexity of the skills measured in the ACT. Because no ACT test form measures all of the skills and knowledge included in the College Readiness Standards, the Standards must be interpreted as skills and knowledge that *most* students who score in a particular score range are *likely* to be able to demonstrate. Since there were relatively few test questions that were answered correctly by 80% or more of the students who scored in the lower score ranges, the Standards in these ranges should be interpreted cautiously. The skills and understandings of students who score in the 1–12 score range may still be evolving. For these students the skills and understandings in the higher score ranges could become their target achievement outcomes.

It is important to recognize that the ACT does not measure everything students have learned nor does any test measure everything necessary for students to know to be successful in college or in the world of work. The ACT Mathematics Test includes questions from a large domain of skills and from areas of knowledge that have been judged important for success in college and beyond. Thus, the College Readiness Standards should be interpreted in a responsible way that will help students understand what they need to know and do if they are going to make a successful transition to college, vocational school, or the world of work. Students can use the Standards to identify the skills and knowledge they need to develop to be better prepared for their future. Teachers and curriculum coordinators can use the Standards to learn more about their students' academic strengths and weaknesses and can then modify their instruction and guide students accordingly.

How Are the College Readiness Standards Organized?

As content experts reviewed the test questions connected to each score range, distinct yet overlapping areas of knowledge and skill were identified. For example, there are many types of questions in which students are asked to solve arithmetic problems. Therefore, *Basic Operations & Applications* is one area, or *strand,* within the College Readiness Standards for ACT Mathematics. The other strands are *Probability, Statistics, & Data Analysis; Numbers: Concepts & Properties; Expressions, Equations, & Inequalities; Graphical Representations; Properties of Plane Figures; Measurement;* and *Functions.*

The strands provide an organizational framework for the College Readiness Standards statements. As you review the Standards, you will note a progression in complexity within each strand. For example, in the 13–15 range for the Basic Operations & Applications strand, students are able to "solve problems in one or two steps using whole numbers," while in the 33–36 range, students demonstrate that they are able to "solve complex arithmetic problems involving percent of increase or decrease and problems requiring integration of several concepts from pre-algebra and/or pre-geometry (e.g., comparing percentages or averages, using several ratios, and finding ratios in geometry settings)."

The Standards are complemented by brief descriptions of learning experiences from which high school students might benefit. Based on the College Readiness Standards, these ideas for progress are designed to provide classroom teachers with help for lesson plan development. These ideas, which are given in Table 1, demonstrate one way that information learned from standardized test results can be used to inform classroom instruction.

Because students learn over time and in various contexts, it is important to use a variety of instructional methods and materials to meet students' diverse needs and to help strengthen and build upon their knowledge and skills. The ideas for progress offer teachers a variety of suggestions to foster learning experiences from which students would likely benefit as they move from one level of learning to the next.

Because learning is a complex and individual process, it is especially important to use multiple sources of information—classroom observations and teacher-developed assessment tools, as well as standardized tests—to accurately reflect what each student knows and can do. The Standards and the ideas for progress, used in conjunction with classroom-based and curricular resources, help teachers and administrators to guide the whole education of every student.

What Are the ACT Mathematics Test College Readiness Standards?

Table 1 on pages 8–15 suggests links between what students are *likely* to be able to do (the College Readiness Standards) and what learning experiences students would likely benefit from.

The College Readiness Standards are organized both by score range (along the left-hand side) and by strand (across the top). The lack of a College Readiness Standards statement in a score range indicates that there was insufficient evidence with which to determine a descriptor.

The ideas for progress are also arranged by score range and by strand. Although many of the ideas cross more than one strand, a primary strand has been identified for each in order to facilitate their use in the classroom. For example, the statement in the 20–23 score range "represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)" brings together concepts from several strands, such as Expressions, Equations, & Inequalities, and Graphical Representations. However, this idea is primarily linked to the Graphical Representations strand.

As you review the table, you will note that ideas for progress have not been provided for the 33–36 score range, the highest score range for the ACT. Students who score in this range on the ACT Mathematics Test have demonstrated proficiency in all, or almost all, of the skills measured by the test. These students will, however, continue to refine and expand their knowledge and skills as they engage in mathematical activities that require critical, logical, and creative thinking.

ACT Mathematics Test

Table 1: The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

Score Range		***Basic Operations & Applications***	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
1–12	**Standards**	■ Students who score in the 1–12 range are most likely beginning to develop the knowledge and skills assessed in the other score ranges.		
	ideas for progress	■ practice and apply estimation and computation using whole numbers and decimals ■ choose the appropriate method of computation to solve multistep problems (e.g., calculator, mental, or pencil and paper) ■ practice selecting appropriate units of measure (e.g., inches or feet, hours or minutes, centimeters or meters) and converting between units ■ model and connect physical, verbal, and symbolic representations of money	■ interpret data from a variety of displays and use it in computation (e.g., mean, median, mode, range) ■ organize, display, and analyze data in a variety of ways	
13–15	**Standards**	■ Perform one-operation computation with whole numbers and decimals ■ Solve problems in one or two steps using whole numbers ■ Perform common conversions (e.g., inches to feet or hours to minutes)	■ Calculate the average of a list of positive whole numbers ■ Perform a single computation using information from a table or chart	■ Recognize equivalent fractions and fractions in lowest terms
	ideas for progress	■ investigate and build understanding of the concept of percentage as a comparison of a part to a whole ■ use multiple operations to solve multistep arithmetic problems	■ solve real-world problems that involve measures of central tendency (e.g., mean, median, mode) ■ interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems ■ conduct simple probability experiments and represent results using different formats	■ recognize and apply place value, rounding, and elementary number theory concepts

Expressions, Equations, & Inequalities	*Graphical Representations*	*Properties of Plane Figures*	*Measurement*	*Functions*
■ model a variety of problem situations with expressions and/or equations ■ use the inverse relationships for the basic operations of addition and subtraction to determine unknown quantities	■ locate and describe points in terms of their position on the number line		■ identify line segments in geometric figures and estimate or calculate their measure	
■ Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) ■ Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals	■ Identify the location of a point with a positive coordinate on the number line		■ Estimate or calculate the length of a line segment based on other lengths given on a geometric figure	
■ use mathematical symbols and variables to express a relationship between quantities (e.g., the number of 59¢ candy bars that you can buy for $5 must satisfy $59n \leq 500$) ■ evaluate algebraic expressions and solve simple equations using integers	■ locate and describe objects in terms of their position on the number line and on a grid	■ describe, compare, and contrast plane and solid figures using their attributes	■ distinguish between area and perimeter, and find the area or perimeter when all relevant dimensions are given	■ recognize functions as mappings of an independent variable into a dependent variable

ACT Mathematics Test

Table 1 (continued): The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
16–19	**Standards**	■ Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent ■ Solve some routine two-step arithmetic problems	■ Calculate the average of a list of numbers ■ Calculate the average, given the number of data values and the sum of the data values ■ Read tables and graphs ■ Perform computations on data from tables and graphs ■ Use the relationship between the probability of an event and the probability of its complement	■ Recognize one-digit factors of a number ■ Identify a digit's place value
	ideas for progress	■ solve routine arithmetic problems that involve rates, proportions, and percents ■ model and solve problems that contain verbal and symbolic representations of money ■ do multistep computations with rational numbers	■ interpret data and use appropriate measures of central tendency to find unknown values ■ find the probability of a simple event in a variety of settings ■ gather, organize, display, and analyze data in a variety of ways to use in problem solving ■ conduct simple probability experiments, use a variety of counting techniques (e.g., Venn diagrams, Fundamental Counting Principle, organized lists), and represent results from data using different formats	■ apply elementary number concepts, including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring ■ recognize, identify, and apply field axioms (e.g., commutative)
20–23	**Standards**	■ Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	■ Calculate the missing data value, given the average and all data values but one ■ Translate from one representation of data to another (e.g., a bar graph to a circle graph) ■ Determine the probability of a simple event ■ Exhibit knowledge of simple counting techniques	■ Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor
	ideas for progress	■ apply and use number properties to model and solve problems that involve reasoning with proportions ■ select and use appropriate units when solving problems that involve one or more units of measure	■ construct and analyze Venn diagrams to help determine simple probabilities	■ use the inverse relationships for the four basic operations, exponentiation, and root extractions to determine unknown quantities ■ perform basic operations with complex numbers

Expressions, Equations, & Inequalities	***Graphical Representations***	***Properties of Plane Figures***	***Measurement***	***Functions***
■ Substitute whole numbers for unknown quantities to evaluate expressions ■ Solve one-step equations having integer or decimal answers ■ Combine like terms (e.g., $2x + 5x$)	■ Locate points on the number line and in the first quadrant	■ Exhibit some knowledge of the angles associated with parallel lines	■ Compute the perimeter of polygons when all side lengths are given ■ Compute the area of rectangles when whole number dimensions are given	
■ create expressions that model mathematical situations using combinations of symbols and numbers ■ evaluate algebraic expressions and solve multistep first-degree equations	■ sketch and identify line segments, midpoints, intersections, and vertical and horizontal lines	■ describe angles and triangles using mathematical terminology and apply their properties	■ find area and perimeter of a variety of polygons by substituting given values into standard geometric formulas	■ evaluate polynomial functions that use function notation ■ distinguish between range and domain
■ Evaluate algebraic expressions by substituting integers for unknown quantities ■ Add and subtract simple algebraic expressions ■ Solve routine first-degree equations ■ Perform straightforward word-to-symbol translations ■ Multiply two binomials	■ Locate points in the coordinate plane ■ Comprehend the concept of length on the number line ■ Exhibit knowledge of slope	■ Find the measure of an angle using properties of parallel lines ■ Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)	■ Compute the area and perimeter of triangles and rectangles in simple problems ■ Use geometric formulas when all necessary information is given	■ Evaluate quadratic functions, expressed in function notation, at integer values
■ identify, interpret, and generate symbolic representations that model the context of a problem ■ factor and perform the basic operations on polynomials ■ create and solve linear equations and inequalities that model real-world situations ■ solve literal equations for any variable	■ represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)	■ recognize what geometric properties and relationships for parallel lines to apply to find unknown angle measures ■ recognize when to apply geometric properties and relationships of triangles to find unknown angle measures	■ apply a variety of strategies to determine the circumference or perimeter and the area for circles, triangles, rectangles, and composite geometric figures	■ identify the basic trigonometric ratios

ACT Mathematics Test

Table 1 (continued): The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
24–27	**Standards**	■ Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	■ Calculate the average, given the frequency counts of all the data values ■ Manipulate data from tables and graphs ■ Compute straightforward probabilities for common situations ■ Use Venn diagrams in counting	■ Find and use the least common multiple ■ Order fractions ■ Work with numerical factors ■ Work with scientific notation ■ Work with squares and square roots of numbers ■ Work problems involving positive integer exponents ■ Work with cubes and cube roots of numbers ■ Determine when an expression is undefined ■ Exhibit some knowledge of the complex numbers
	ideas for progress	■ model and solve real-world problems that involve a combination of rates, proportions, and/or percents	■ find the probability of simple events, disjoint events, compound events, and independent events in a variety of settings using a variety of counting techniques	■ apply and use elementary number concepts and number properties to model and solve nonroutine problems that involve new ideas

Expressions, Equations, & Inequalities	***Graphical Representations***	***Properties of Plane Figures***	***Measurement***	***Functions***
■ Solve real-world problems using first-degree equations ■ Write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) ■ Identify solutions to simple quadratic equations ■ Add, subtract, and multiply polynomials ■ Factor simple quadratics (e.g., the difference of squares and perfect square trinomials) ■ Solve first-degree inequalities that do not require reversing the inequality sign	■ Identify the graph of a linear inequality on the number line ■ Determine the slope of a line from points or equations ■ Match linear graphs with their equations ■ Find the midpoint of a line segment	■ Use several angle properties to find an unknown angle measure ■ Recognize Pythagorean triples ■ Use properties of isosceles triangles	■ Compute the area of triangles and rectangles when one or more additional simple steps are required ■ Compute the area and circumference of circles after identifying necessary information ■ Compute the perimeter of simple composite geometric figures with unknown side lengths	■ Evaluate polynomial functions, expressed in function notation, at integer values ■ Express the sine, cosine, and tangent of an angle in a right triangle as a ratio of given side lengths
■ create and use basic families of functions (which include linear, absolute value, and quadratic) to model and solve problems in common settings ■ explore and use different methods to solve systems of equations ■ manipulate radical expressions (e.g., rationalize denominators)	■ graph linear equations and inequalities, determine slopes of lines, identify parallel and perpendicular lines, and find distances ■ identify characteristics of figures from a general equation	■ apply special right-triangle properties and the Pythagorean theorem to solve congruent and similar shape problems	■ apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures	■ write an expression for and evaluate composite functions ■ use basic trigonometric ratios to solve problems involving indirect measurement

ACT MATHEMATICS TEST

Table 1 (continued): The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
28–32	**Standards**	■ Solve word problems containing several rates, proportions, or percentages	■ Calculate or use a weighted average ■ Interpret and use information from figures, tables, and graphs ■ Apply counting techniques ■ Compute a probability when the event and/or sample space are not given or obvious	■ Apply number properties involving prime factorization ■ Apply number properties involving even/odd numbers and factors/multiples ■ Apply number properties involving positive/negative numbers ■ Apply rules of exponents ■ Multiply two complex numbers
	ideas for progress	■ solve problems that require combining multiple concepts	■ design and conduct probability investigations (e.g., how the margin of error is determined) and then determine, analyze, and communicate the results	■ explain, solve, and/or draw conclusions for complex problems using relationships and elementary number concepts
33–36	**Standards**	■ Solve complex arithmetic problems involving percent of increase or decrease and problems requiring integration of several concepts from pre-algebra and/or pre-geometry (e.g., comparing percentages or averages, using several ratios, and finding ratios in geometry settings)	■ Distinguish between mean, median, and mode for a list of numbers ■ Analyze and draw conclusions based on information from figures, tables, and graphs ■ Exhibit knowledge of conditional and joint probability	■ Draw conclusions based on number concepts, algebraic properties, and/or relationships between expressions and numbers ■ Exhibit knowledge of logarithms and geometric sequences ■ Apply properties of complex numbers

Expressions, Equations, & Inequalities	*Graphical Representations*	*Properties of Plane Figures*	*Measurement*	*Functions*
■ Manipulate expressions and equations ■ Write expressions, equations, and inequalities for common algebra settings ■ Solve linear inequalities that require reversing the inequality sign ■ Solve absolute value equations ■ Solve quadratic equations ■ Find solutions to systems of linear equations	■ Interpret and use information from graphs in the coordinate plane ■ Match number line graphs with solution sets of linear inequalities ■ Use the distance formula ■ Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point ■ Recognize special characteristics of parabolas and circles (e.g., the vertex of a parabola and the center or radius of a circle)	■ Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles ■ Use the Pythagorean theorem	■ Use relationships involving area, perimeter, and volume of geometric figures to compute another measure	■ Evaluate composite functions at integer values ■ Apply basic trigonometric ratios to solve right-triangle problems
■ formulate expressions, equations, and inequalities that require planning to accurately model real-world problems (e.g., direct and inverse variation)	■ solve and graph quadratic inequalities	■ make generalizations, arrive at conclusions based on conditional statements, and offer solutions for new situations that involve connecting mathematics with other content areas ■ investigate angle and arc relationships for circles	■ examine and compare a variety of methods to find areas of composite figures and construct scale drawings	■ explore geometric models where unit circle trigonometry and basic identities can be used to solve problems
■ Write expressions that require planning and/or manipulating to accurately model a situation ■ Write equations and inequalities that require planning, manipulating, and/or solving ■ Solve simple absolute value inequalities	■ Match number line graphs with solution sets of simple quadratic inequalities ■ Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$ ■ Solve problems integrating multiple algebraic and/or geometric concepts ■ Analyze and draw conclusions based on information from graphs in the coordinate plane	■ Draw conclusions based on a set of conditions ■ Solve multistep geometry problems that involve integrating concepts, planning, visualization, and/or making connections with other content areas ■ Use relationships among angles, arcs, and distances in a circle	■ Use scale factors to determine the magnitude of a size change ■ Compute the area of composite geometric figures when planning or visualization is required	■ Write an expression for the composite of two simple functions ■ Use trigonometric concepts and basic identities to solve problems ■ Exhibit knowledge of unit circle trigonometry ■ Match graphs of basic trigonometric functions with their equations

Description of the ACT Mathematics Test

What Does the ACT Mathematics Test Measure?

The ACT Mathematics Test is a 60-question, 60-minute test designed to assess the mathematical reasoning skills that students have typically acquired in courses taken up to the beginning of Grade 12. The multiple-choice test requires students to analyze problems in real-world and purely mathematical settings, plan and carry out solution strategies, and verify the appropriateness of solutions. Most of the test questions are individual items, but some may belong to sets (i.e., several items based on the same graph, chart, or information).

On the ACT Mathematics Test, students demonstrate their ability to read and understand mathematical terminology; to apply definitions, algorithms, theorems, and properties; to interpret and analyze data; and to use mathematics to solve problems.

Students also apply quantitative reasoning in a variety of ways, such as discerning relationships between mathematical concepts, connecting and integrating mathematical concepts and ideas, and making generalizations. Computational skills and knowledge of basic formulas are assumed as background for the problems, but extensive computation and memorization of complex formulas are not required. The concepts covered on the test emphasize the major content areas that are prerequisite to successful performance in entry-level college mathematics courses.

The questions focus on mathematical reasoning and making connections within and among six content areas and at various cognitive levels. These areas and levels are shown in Figure 1 on page 17.

Through the various cognitive levels, students demonstrate their ability to use and reason with mathematics. *Knowledge and Skills* questions (about 50% of the test) require students to use one or more facts, definitions, formulas, or procedures to solve problems that are presented in purely mathematical terms. *Direct Application* questions (about 28% of the test) require students to use their knowledge and skills to solve straightforward problems set in real-world situations. *Understanding Concepts* and *Integrating Conceptual Understanding* questions (about 22% of the test) assess students' depth of understanding of major concepts by requiring reasoning from a single concept or the integration of several concepts to reach an inference or a conclusion.

The content of the ACT Mathematics Test is reflective of the content taught in mathematics classrooms and of the prerequisite skills and understandings necessary for successful transition to entry-level college mathematics courses. ACT routinely monitors the high school mathematics curriculum through reviews of state and national standards, current textbooks, and national organizations' curriculum frameworks; surveys of secondary and postsecondary instructors; and meetings with education consultants. A brief description of the content sampled on the test and the approximate percentage of the test devoted to each content area on the ACT Mathematics Test are provided on page 17.

> **"The test should measure what students can do with what they have learned."**
>
> — (ACT, 1996b, p. 2)

Pre-Algebra (23%). Questions in this content area are based on basic operations using whole numbers, decimals, fractions, and integers; place value; square roots and approximations; the concept of exponents; scientific notation; factors; ratio, proportion, and percent; linear equations in one variable; absolute value and ordering numbers by value; elementary counting techniques and simple probability; data collection, representation, and interpretation; and understanding simple descriptive statistics.

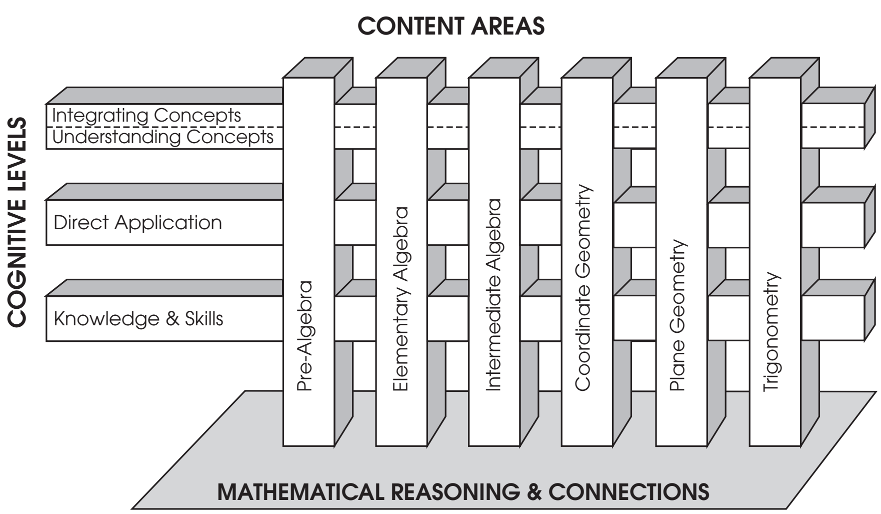
Elementary Algebra (17%). Questions in this content area are based on properties of exponents and square roots, evaluation of algebraic expressions through substitution, using variables to express functional relationships, understanding algebraic operations, and solutions of quadratic equations by factoring.

Intermediate Algebra (15%). Questions in this content area are based on an understanding of the quadratic formula, rational and radical expressions, absolute value equations and inequalities, sequences and patterns, systems of equations, quadratic inequalities, functions, modeling, matrices, roots of polynomials, and complex numbers.

Coordinate Geometry (15%). Questions in this content area are based on graphing and the relations between equations and graphs, including points, lines, polynomials, circles, and other curves; graphing inequalities; slope; parallel and perpendicular lines; distance; midpoints; and conics.

Plane Geometry (23%). Questions in this content area are based on the properties and relations of plane figures, including angles and relations among perpendicular and parallel lines; properties of circles, triangles, rectangles, parallelograms, and trapezoids; transformations; the concept of proof and proof techniques; volume; and applications of geometry to three dimensions.

Trigonometry (7%). Questions in this content area are based on understanding trigonometric functions; graphing trigonometric functions; modeling using trigonometric functions; use of trigonometric identities; and solving trigonometric equations.



Adapted from *Mathematics Framework for the 1996 National Assessment of Education Progress* (p.11)

Figure 1: **ACT Mathematics Test Content Areas and Cognitive Levels**

What Is ACT's Calculator Policy for the ACT?

Students may use any four-function, any scientific, and almost any graphing calculator on the ACT Mathematics Test. However, calculators are **not required.** All problems can be solved without a calculator. If students regularly use a calculator in their math work, they are encouraged to use one they are familiar with as they take the Mathematics Test. Using a more powerful, but unfamiliar, calculator is not likely to give students an advantage over using the kind they normally use.

How Are the Test Questions Linked to the College Readiness Standards?

The ACT Mathematics Test assesses various kinds and combinations of skills; each of these skills can be measured in different ways. You may have noticed that the strands and the content areas are not the same. The strands are areas in which there are a variety of test questions representing a continuum of skills and understandings. The strands are similar to those found in state and national frameworks. Many of the strands cut across the different content areas on the ACT Mathematics Test.

Table 2 below provides the strands and the corresponding content areas.

Table 2: **ACT Mathematics Strands and Corresponding Content Areas**

Strand	**Content Area**
Basic Operations & Applications	Pre-Algebra
Probability, Statistics, & Data Analysis	Pre-Algebra Elementary Algebra
Numbers: Concepts & Properties	Pre-Algebra Elementary Algebra Intermediate Algebra
Expressions, Equations, & Inequalities	Pre-Algebra Elementary Algebra Intermediate Algebra
Graphical Representations	Coordinate Geometry
Properties of Plane Figures	Plane Geometry
Measurement	Plane Geometry Coordinate Geometry
Functions	Intermediate Algebra Coordinate Geometry Trigonometry

The Need for Thinking Skills

Every student comes to school with the ability to think, but to achieve their goals students need to develop skills such as learning to make new connections between texts and ideas, to understand increasingly complex concepts, and to think through their assumptions. Because of technological advances and the fast pace of our society, it is increasingly important that students not only know information but also know how to critique and manage that information. Students must be provided with the tools for ongoing learning; understanding, analysis, and generalization skills must be developed so that the learner is able to adapt to a variety of situations.

How Are ACT Test Questions Linked to Thinking Skills?

Our belief in the importance of developing thinking skills in learners was a key factor in the development of the ACT. ACT believes that students' preparation for further learning is best assessed by measuring, as directly as possible, the academic skills that students have acquired and that they will need to perform at the next level of learning. The required academic skills can most directly be assessed by reproducing as faithfully as possible the complexity of the students' schoolwork. Therefore, the ACT test questions are designed to determine how skillfully students solve problems, grasp implied meanings, draw inferences, evaluate ideas, and make judgments in subject-matter areas important to success in intellectual work both inside and outside school.

Table 3 on pages 20–33 provides sample test questions, organized by score range, that are linked to specific skills within each of the eight Mathematics strands. It is important to note the increasing level of skill with mathematics—computing, reasoning, and making connections—that students scoring in the higher score ranges are able to demonstrate. The questions were chosen to illustrate the variety of content as well as the range of complexity within each strand. The sample test questions for the 16–19, 20–23, 24–27, 28–32, and 33–36 score ranges are examples of items answered correctly by 80% or more of the ACT examinees who obtained scores in each of these five score ranges. The sample test questions for the 13–15 score range, however, are examples of items answered correctly by 80% of the PLAN examinees who obtained scores in this score range. PLAN test questions are given for the 13–15 score range because it was not possible, using the 80% criterion described on page 5, to identify ACT sample test questions for this score range.

As you review the sample test questions, you will note that each correct answer is marked with an asterisk. For score ranges that include more than one skill, boldface type is used to denote the skill that corresponds to the sample test question.

"Learning is not attained by chance, it must be sought for with ardour and attended to with diligence."

— Abigail Adams in a letter to John Quincy Adams

Table 3: **ACT Sample Test Questions by Score Range**
Basic Operations & Applications Strand

Score Range	***Basic Operations & Applications***	***Sample Test Questions***
13–15	Perform one-operation computation with whole numbers and decimals **Solve problems in one or two steps using whole numbers** Perform common conversions (e.g., inches to feet or hours to minutes)	Ten boxes of books were delivered to the school library. There were 50 books in each box, except for the last box, which contained only 40 books. How many books did the library receive in this delivery? **A.** 50 **B.** 450 ***C.** 490 **D.** 500 **E.** 540
16–19	Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent **Solve some routine two-step arithmetic problems**	A *stone* is a unit of weight equivalent to 14 pounds. If a person weighs 177 pounds, how many stone, to the nearest tenth, does this person weigh? **A.** 247.8 **B.** 126.4 **C.** 79.1 ***D.** 12.6 **E.** 7.9
20–23	Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	Near a large city, planes take off from two airfields. One of the fields is capable of sending up a plane every 3 minutes. The other field is capable of sending up 2 planes every 7 minutes. At these rates, which of the following is the most reasonable estimate of the total number of planes the two airfields could send up in 90 minutes? **A.** 18 **B.** 27 **C.** 36 **D.** 44 ***E.** 55
24–27	Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	Four students about to purchase concert tickets for $18.50 for each ticket discover that they may purchase a block of 5 tickets for $80.00. How much would each of the 4 save if they can get a fifth person to join them and the 5 people equally divide the price of the 5-ticket block? **A.** $ 1.50 ***B.** $ 2.50 **C.** $ 3.13 **D.** $10.00 **E.** $12.50
28–32	Solve word problems containing several rates, proportions, or percentages	A performance was rated on a 3-point scale by an audience. A rating of 1 was given by 30% of the audience, a rating of 2 by 60%, and a rating of 3 by 10%. To the nearest tenth, what was the average of the ratings? **A.** 1.2 **B.** 1.5 ***C.** 1.8 **D.** 2.0 **E.** 2.2

Table 3: **ACT Sample Test Questions by Score Range**
Basic Operations & Applications Strand

Score Range	***Basic Operations & Applications***	***Sample Test Questions***
33–36	Solve complex arithmetic problems involving percent of increase or decrease and problems requiring integration of several concepts from pre-algebra and/or pre-geometry (e.g., comparing percentages or averages, using several ratios, and finding ratios in geometry settings)	Yvette earned a score of 56 on a recent 25-question multiple-choice exam. The scoring for the exam was +6 for each correct answer, –2 for each incorrect answer, and 0 for each unanswered question. What is the *maximum* number of questions Yvette could have answered correctly? **A.** 9 **B.** 10 **C.** 11 ***D.** 13 **E.** 14

Table 3: **ACT Sample Test Questions by Score Range**
Probability, Statistics, & Data Analysis Strand

Score Range	***Probability, Statistics, & Data Analysis***	***Sample Test Questions***
13–15	Calculate the average of a list of positive whole numbers **Perform a single computation using information from a table or chart**	Kay's Appliance Repair Shop charges for labor according to the chart below. What is the charge for each additional 15 minutes of labor beyond the initial 30 minutes? minutes of labor: 1–30, 31–45, 46–60, 61–75, 76–90 charge: $39.90, $49.35, $58.80, $68.25, $77.70 **A.** $4.73 **B.** $6.30 **C.** $7.09 **D.** $7.56 ***E.** $9.45
16–19	**Calculate the average of a list of numbers** Calculate the average, given the number of data values and the sum of the data values Read tables and graphs Perform computations on data from tables and graphs Use the relationship between the probability of an event and the probability of its complement	Contributions to a charity are made by each of 5 companies as indicated in the table below. Company: A, B, C, D, E Contribution in dollars: 0, 300, 300, 180, 270 What is the average of the contributions made by the 5 companies? **A.** $187.50 ***B.** $210.00 **C.** $250.00 **D.** $262.50 **E.** $350.00
20–23	Calculate the missing data value, given the average and all the data values but one Translate from one representation of data to another (e.g., a bar graph to a circle graph) Determine the probability of a simple event **Exhibit knowledge of simple counting techniques**	Rudi has 5 pairs of slacks, 6 blouses, and 2 sweaters in her closet. How many different outfits, composed of a pair of slacks, a blouse, and a sweater, can she choose from this closet? **A.** 2 **B.** 6 **C.** 13 **D.** 32 ***E.** 60
24–27	Calculate the average, given the frequency counts of all the data values Manipulate data from tables and graphs **Compute straightforward probabilities for common situations** Use Venn diagrams in counting	Of the 50 people in a music store, 35 of them own a cassette tape player. If 1 of those 50 people is randomly selected to win a cassette tape, what is the probability that the person chosen will already own a cassette tape player? ***A.** .70 **B.** .65 **C.** .35 **D.** .30 **E.** .15

Table 3: **ACT Sample Test Questions by Score Range**
Probability, Statistics, & Data Analysis Strand

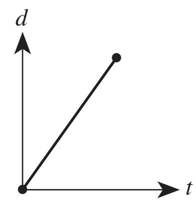
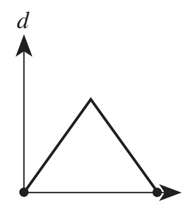
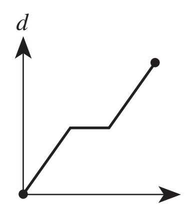
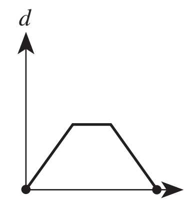
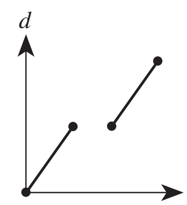
Score Range	***Probability, Statistics, & Data Analysis***	***Sample Test Questions***
28–32	Calculate or use a weighted average Interpret and use information from figures, tables, and graphs **Apply counting techniques** Compute a probability when the event and/or sample space are not given or obvious	How many possible combinations of $1 and/or $5 bills could be in a cash register containing exactly $20, in $1 and/or $5 bills? **A.** 3 **B.** 4 ***C.** 5 **D.** 10 **E.** 20
33–36	Distinguish between mean, median, and mode for a list of numbers **Analyze and draw conclusions based on information from figures, tables, and graphs** Exhibit knowledge of conditional and joint probability	Starting at her doorstep, Ramona walked down the sidewalk at 1.5 feet per second for 4 seconds. Then she stopped for 4 seconds, realizing that she had forgotten something. Next she returned to her doorstep along the same route at 1.5 feet per second. The graph of Ramona's distance (d) from her doorstep as a function of time (t) would most resemble which of the following? **A.**  **B.**  **C.**  ***D.**  **E.** 

Table 3: **ACT Sample Test Questions by Score Range**
Numbers: Concepts & Properties Strand

Score Range	***Numbers: Concepts & Properties***	***Sample Test Questions***
13–15	Recognize equivalent fractions and fractions in lowest terms	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
16–19	Recognize one-digit factors of a number **Identify a digit's place value**	The greatest common divisor of 84, 90, and 66 (that is, the largest exact divisor of all three numbers) is: ***A.** 6 **B.** 12 **C.** 18 **D.** 36 **E.** 90
20–23	Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor	What is the 7th term in this sequence of "triangular" numbers, defined by the figures below: 1, 3, 6, 10, . . . ? * * * * * * * * * * * * * * * * * * * * . . . 1 3 6 10 **A.** 7 **B.** 22 **C.** 25 ***D.** 28 **E.** 40
24–27	**Find and use the least common multiple** Order fractions Work with numerical factors Work with scientific notation Work with squares and square roots of numbers Work problems involving positive integer exponents Work with cubes and cube roots of numbers Determine when an expression is undefined Exhibit some knowledge of the complex numbers	What is the least common multiple of 80, 70, and 90 ? **A.** 80 **B.** 240 **C.** 504 ***D.** 5,040 **E.** 504,000
28–32	Apply number properties involving prime factorization Apply number properties involving even/odd numbers and factors/multiples **Apply number properties involving positive/negative numbers** Apply rules of exponents Multiply two complex numbers	If a and b are real numbers, and $a > b$ and $b < 0$, then which of the following inequalities must be true? **A.** $a > 0$ **B.** $a < 0$ **C.** $a^2 > b^2$ **D.** $a^2 < b^2$ ***E.** $b^2 > 0$

Table 3: ACT Sample Test Questions by Score Range
Numbers: Concepts & Properties Strand

Score Range	Numbers: Concepts & Properties	Sample Test Questions
33–36	**Draw conclusions based on number concepts, algebraic properties, and/or relationships between expressions and numbers** Exhibit knowledge of logarithms and geometric sequences Apply properties of complex numbers	If $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$, which of the following *must* be true? **A.** $a = b$ **B.** $a + b < 0$ ***C.** $a = 0$ or $b = 0$ **D.** $a + b > 1$ **E.** $a = 1$ and $b > 0$

Table 3:	**ACT Sample Test Questions by Score Range** *Expressions, Equations, & Inequalities* Strand	
Score Range	***Expressions, Equations, & Inequalities***	***Sample Test Questions***
13–15	Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) **Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals**	What is the solution of $x + 3.4 = 20.91$? **A.** 24.31 **B.** 23.95 **C.** 17.87 ***D.** 17.51 **E.** 6.15
16–19	**Substitute whole numbers for unknown quantities to evaluate expressions** Solve one-step equations having integer or decimal answers Combine like terms (e.g., $2x + 5x$)	The operation $x \sim y$ stands for $\frac{x+y}{x-y}$. Which of the following is equal to $7 \sim 3$? **A.** 10 **B.** 6 **C.** $5\frac{1}{4}$ **D.** 4 ***E.** $2\frac{1}{2}$
20–23	Evaluate algebraic expressions by substituting integers for unknown quantities Add and subtract simple algebraic expressions **Solve routine first-degree equations** Perform straightforward word-to-symbol translations Multiply two binomials	If $-(5x - 21) = 2x$, then x = ? ***A.** 3 **B.** 5 **C.** 7 **D.** –3 **E.** –7
24–27	**Solve real-world problems using first-degree equations** Write expressions, equations or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) Identify solutions to simple quadratic equations Add, subtract, and multiply polynomials Factor simple quadratics (e.g., the difference of squares and perfect square trinomials) Solve first-degree inequalities that do not require reversing the inequality sign	The relationship between temperature expressed in degrees Fahrenheit (F) and degrees Celsius (C) is given by the formula $F = \frac{9}{5}C + 32$ If the temperature is 14 degrees Fahrenheit, what is it in degrees Celsius? ***A.** –10 **B.** –12 **C.** –14 **D.** –16 **E.** –18

Table 3: **ACT Sample Test Questions by Score Range**
Expressions, Equations, & Inequalities Strand

Score Range	***Expressions, Equations, & Inequalities***	***Sample Test Questions***
28–32	Manipulate expressions and equations Write expressions, equations, and inequalities for common algebra settings Solve linear inequalities that require reversing the inequality sign Solve absolute value equations **Solve quadratic equations** Find solutions to systems of linear equations	The equation $10w^2 + 17w - 20 = 0$ has what types of numbers as its two solutions? **A.** Two negative real numbers ***B.** One positive real number and one negative real number **C.** Two positive real numbers **D.** One negative real number and zero **E.** One positive real number and zero
33–36	**Write expressions that require planning and/or manipulating to accurately model a situation** Write equations and inequalities that require planning, manipulating, and/or solving Solve simple absolute value inequalities	If during 1 hour of a certain television program there are $x + y$ commercials, where x of them are 30-second commercials and the rest are 1-minute commercials, which of the following expressions represents the number of minutes left for noncommercial programming during the hour? **A.** $60 - 2x - y$ **B.** $60 - 30x - y$ **C.** $60 - 30x - 60y$ ***D.** $60 - \frac{1}{2}x - y$ **E.** $60 - \frac{1}{30}x - \frac{1}{60}y$

Table 3: **ACT Sample Test Questions by Score Range**
Graphical Representations Strand

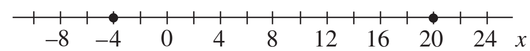
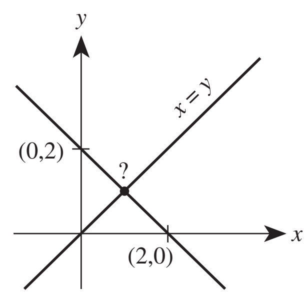
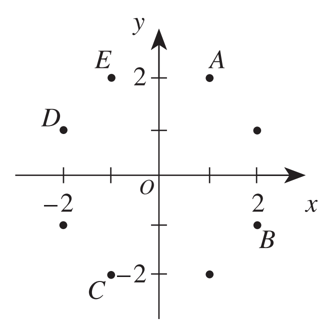
Score Range	***Graphical Representations***	***Sample Test Questions***
13–15	Identify the location of a point with a positive coordinate on the number line	The coordinates of the endpoints of a certain segment on the real number line below are –4 and 20. What is the coordinate of the midpoint of this segment?  **A.** 7 ***B.** 8 **C.** 10 **D.** 12 **E.** 16
16–19	Locate points on the number line and in the first quadrant	Two straight lines are graphed in the standard (x,y) coordinate plane below. What are the coordinates of their intersection?  **A.** $\left(\frac{3}{4},\frac{3}{4}\right)$ ***B.** $(1,1)$ **C.** $\left(\frac{3}{2},\frac{3}{2}\right)$ **D.** $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ **E.** $\left(\sqrt{2},\sqrt{2}\right)$
20–23	**Locate points in the coordinate plane** Comprehend the concept of length on the number line Exhibit knowledge of slope	Eight points with integer coordinates are plotted in the standard (x,y) coordinate plane below. Which of the plotted points has an x-coordinate less than 1 and a y-coordinate of at least 2 ?  **A.** *A* **B.** *B* **C.** *C* **D.** *D* ***E.** *E*
24–27	Identify the graph of a linear inequality on the number line **Determine the slope of a line from points or equations** Match linear graphs with their equations Find the midpoint of a line segment	What is the slope of the line $4x - 3y = 5$? **A.** $-\frac{5}{3}$ **B.** $-\frac{4}{3}$ **C.** $-\frac{3}{4}$ **D.** $\frac{3}{4}$ ***E.** $\frac{4}{3}$

Table 3: **ACT Sample Test Questions by Score Range**
Graphical Representations Strand

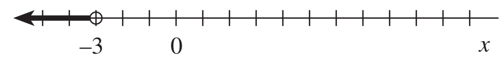
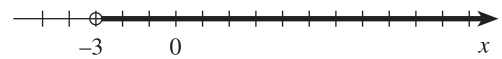
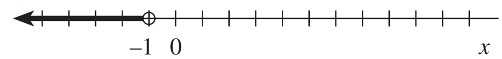
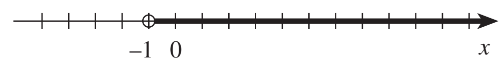
Score Range	***Graphical Representations***	***Sample Test Questions***
28–32	Interpret and use information from graphs in the coordinate plane **Match number line graphs with solution sets of linear inequalities** Use the distance formula Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point Recognize special characteristics of parabolas and circles (e.g., the vertex of a parabola and the center or radius of a circle)	Which of the following is the graph of the solution set for $3(2 + x) < 3$? **A.**  **B.**  ***C.**  **D.**  **E.** 
33–36	Match number line graphs with solution sets of simple quadratic inequalities **Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$** Solve problems integrating multiple algebraic and/or geometric concepts Analyze and draw conclusions based on information from graphs in the coordinate plane	The equation $y = P(x)$ is graphed in the standard (x,y) coordinate plane. If $P(x)$ is a 5th degree polynomial, which of the following CANNOT be the number of times the graph intersects (touches or crosses) the x-axis? ***A.** 0 **B.** 1 **C.** 2 **D.** 3 **E.** 5

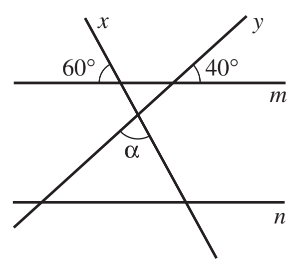
Table 3:	**ACT Sample Test Questions by Score Range** *Properties of Plane Figures* Strand	
Score Range	***Properties of Plane Figures***	***Sample Test Questions***
13–15		
16–19	Exhibit some knowledge of the angles associated with parallel lines	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
20–23	Find the measure of an angle using properties of parallel lines **Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)**	If one angle in a triangle measures 18° and another measures 36°, what is the measure of the third angle? **A.** 36° **B.** 46° **C.** 54° ***D.** 126° **E.** 144°
24–27	**Use several angle properties to find an unknown angle measure** Recognize Pythagorean triples Use properties of isosceles triangles	Lines *m* and *n* below are parallel, and lines *x* and *y* are transversals. What is the value of α ? **A.** 60° **B.** 70° ***C.** 80° **D.** 100° **E.** 110° 
28–32	**Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles** Use the Pythagorean theorem	If the lengths of the sides of one triangle are 8 inches, 10 inches, and 12 inches, respectively, what is the perimeter, in inches, of a similar triangle whose longest side is 4 inches? **A.** 90 **B.** 30 **C.** 15 **D.** 12 ***E.** 10
33–36	Draw conclusions based on a set of conditions **Solve multistep geometry problems that involve integrating concepts, planning, visualization, and/or making connections with other content areas** Use relationships among angles, arcs, and distances in a circle	An object detected on radar is 5 miles to the east, 4 miles to the north, and 1 mile above the tracking station. Among the following, which is the closest approximation to the distance, in miles, that the object is from the tracking station? ***A.** 6.5 **B.** 7.2 **C.** 8.3 **D.** 9.0 **E.** 10.0

Table 3: **ACT Sample Test Questions by Score Range**
Measurement Strand

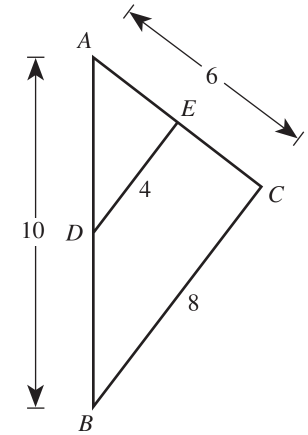
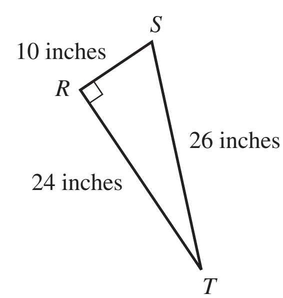
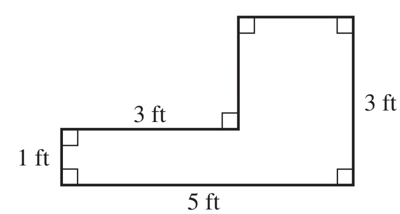
Score Range	***Measurement***	***Sample Test Questions***
13–15	Estimate or calculate the length of a line segment based on other lengths given on a geometric figure	In the figure below, the lengths of line segments are given in feet. If $\overline{BC}$ is parallel to $\overline{DE}$, how many feet long is $\overline{AE}$? **A.** $2\sqrt{21}$ **B.** $2\frac{2}{5}$ ***C.** 3 **D.** $6\frac{2}{3}$ **E.** 12 
16–19	**Compute the perimeter of polygons when all side lengths are given** Compute the area of rectangles when whole number dimensions are given	The out-of-bounds lines around a basketball court in Central Park need to be repainted. The court is a rectangle 90 feet long and 50 feet wide. What is its perimeter, in feet? **A.** 140 **B.** 190 **C.** 230 ***D.** 280 **E.** 4,500
20–23	**Compute the area and perimeter of triangles and rectangles in simple problems** Use geometric formulas when all necessary information is given	In the triangle below, $\angle R$ is a right angle and the lengths of the sides are as marked. In square inches, what is the area of $\triangle RST$? **A.** 60 ***B.** 120 **C.** 130 **D.** 240 **E.** 312 
24–27	Compute the area of triangles and rectangles when one or more additional simple steps are required Compute the area and circumference of circles after identifying necessary information **Compute the perimeter of simple composite geometric figures with unknown side lengths**	How many feet long is the perimeter of the figure sketched below? **A.** 12 **B.** 14 **C.** 15 ***D.** 16 **E.** 18 

Table 3: **ACT Sample Test Questions by Score Range**
Measurement Strand

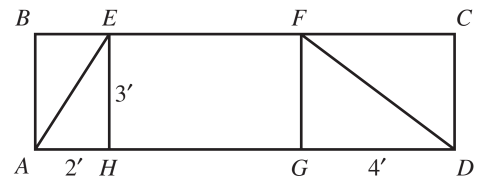
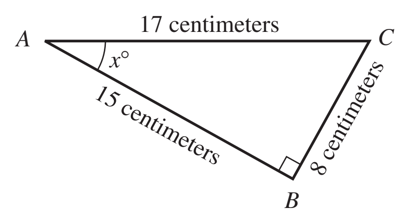
Score Range	***Measurement***	***Sample Test Questions***
28–32	Use relationships involving area, perimeter, and volume of geometric figures to compute another measure	Distances marked on the figure below are in feet. Points B, E, F, and C are collinear as are points A, H, G, and D. If the area of rectangle $ABCD$ is 33 square feet and $\overline{EH}$ and $\overline{FG}$ are each perpendicular to $\overline{AD}$, what is the area, in square feet, of trapezoid $AEFD$?  **A.** 15 **B.** 21 ***C.** 24 **D.** 27 **E.** 48
33–36	**Use scale factors to determine the magnitude of a size change** Compute the area of composite geometric figures when planning or visualization is required	There are 3 empty spherical containers with radii of 1, 2, and 4 meters, respectively. How many fillings of the middle-sized container would be equivalent to 1 filling of the largest container? (Note: The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.) **A.** $\frac{1}{2}$ **B.** 2 **C.** 4 **D.** 7 ***E.** 8

Table 3: **ACT Sample Test Questions by Score Range**
Functions Strand

<table>
<tr><th>Score Range</th><th>Functions</th><th>Sample Test Questions</th></tr>
<tr><td>13–15</td><td></td><td></td></tr>
<tr><td>16–19</td><td></td><td></td></tr>
<tr><td>20–23</td><td>Evaluate quadratic functions, expressed in function notation, at integer values</td><td>If $f(x) = -2x^2$, then $f(-3) = ?$

A. –36
*B. –18
C. 12
D. 18
E. 36</td></tr>
<tr><td>24–27</td><td>Evaluate polynomial functions, expressed in function notation, at integer values

Express the sine, cosine, and tangent of an angle in a right triangle as a ratio of given side lengths</td><td>In the figure below, $\angle B$ is a right angle and other measures are marked. What is tan x ?



A. $\frac{15}{8}$
*B. $\frac{8}{15}$
C. $\frac{8}{17}$
D. $\frac{15}{17}$
E. $\frac{17}{15}$</td></tr>
<tr><td>28–32</td><td>Evaluate composite functions at integer values

Apply basic trigonometric ratios to solve right-triangle problems</td><td>For the 2 functions $f(x)$ and $g(x)$, tables of values are shown below. What is the value of $g\left(f(3)\right)$?

<table><tr><th>x</th><th>$f(x)$</th></tr><tr><td>–5</td><td>7</td></tr><tr><td>–2</td><td>–5</td></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>2</td></tr></table>
<table><tr><th>x</th><th>$g(x)$</th></tr><tr><td>–2</td><td>3</td></tr><tr><td>1</td><td>–1</td></tr><tr><td>2</td><td>–3</td></tr><tr><td>3</td><td>–5</td></tr></table>
A. –5
*B. –3
C. –1
D. 2
E. 7</td></tr>
<tr><td>33–36</td><td>Write an expression for the composite of two simple functions

Use trigonometric concepts and basic identities to solve problems

Exhibit knowledge of unit circle trigonometry

Match graphs of basic trigonometric functions with their equations</td><td>Which of the following is equivalent to $\frac{1 - \cos^2\theta}{\cos^2\theta}$?

A. $\sec^2\theta$
B. $(\csc^2\theta) - 1$
*C. $\tan^2\theta$
D. $\sin^2\theta$
E. $-\frac{1}{\sin^2\theta}$</td></tr>
</table>

Thinking Your Way Through the ACT Test

In our increasingly complex society, students' ability to think critically and make informed decisions is more important than ever. The workplace demands new skills and knowledge and continual learning; information bombards consumers through media and the Internet; familiar assumptions and values often come into question. More than ever before, students in today's classrooms face a future when they will need to adapt quickly to change, to think about issues in rational and creative ways, to cope with ambiguities, and to find means of applying information to new situations.

Classroom teachers are integrally involved in preparing today's students for their futures. Such preparation must include the development of thinking skills such as problem solving, decision making, and inferential and evaluative thinking. These are, in fact, the types of skills and understandings that underlie the test questions on the ACT.

How Can Analyzing Test Questions Build Thinking Skills?

On pages 35–36, you will find additional sample test questions. The sample test questions provide a link to a strand, a Standard, and a score range. Each sample test question includes a description of the skills and understandings students must demonstrate in order to arrive at the correct answer. The descriptions provide a series of strategies students typically might employ as they work through each test question. Possible flawed strategies leading to the choice of one or more incorrect responses also are offered. Analyzing test questions in this way, as test developers do to produce a Test Question Rationale, can provide students with a means of understanding the knowledge and skills embedded in the test questions and an opportunity to explore why an answer choice is correct or incorrect.

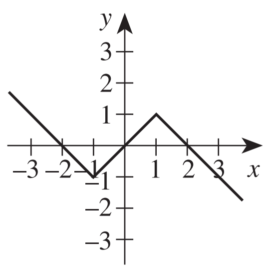
Providing students with strategies such as these encourages them to take charge of their thinking and learning. The sample test questions that appear in Table 3 on pages 20–33 can be used to develop additional Test Question Rationales.

"Learning is fundamentally about *making and maintaining connections* . . . among concepts, ideas, and meanings."

— American Association for Higher Education, American College Personnel Association, & National Association of Student Personnel Administrators, June 1998

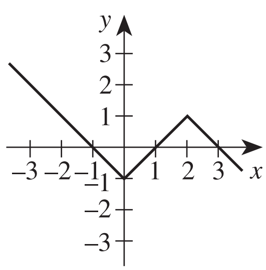
Test Question Rationale	
Graphical Representations	■ Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$ ■ 33–36 score range

1. The graph of $y = f(x)$ is shown below.

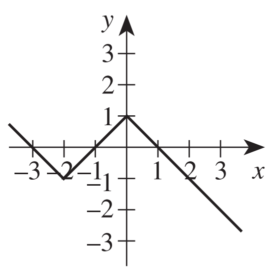


Which of the following is the graph of $y = f(x) + 1$?

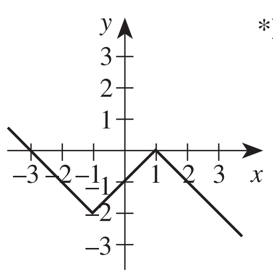
A.



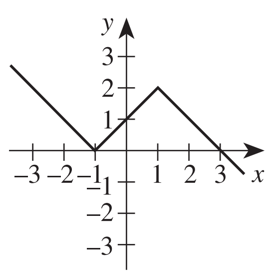
D.



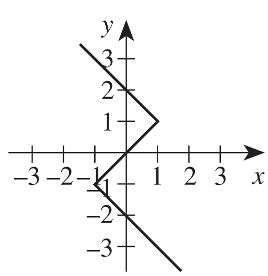
B.



***E.**



C.

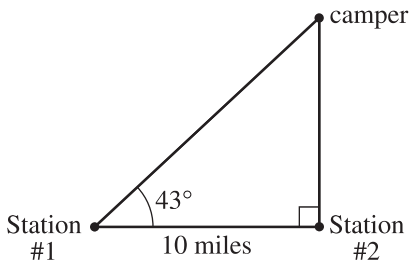


Question 1 requires that students understand underlying concepts of functions. This problem asks students to work with a function where the algebraic form of the function is not a concrete equation. Students have to make graphical connections to the functional form $f(x) + 1$. A student might reason that, for each particular x-value, the new y-value is 1 more than the old y-value, which was $f(x)$, and so the graph would shift up by 1 unit at each x-value. The result of this reasoning would yield the correct graph, answer choice E.

Students who suspect that the new graph is a translation of the old one but are not sure of the direction might be likely to select answer choice A, a popular choice. A student who was unsure of the direction of the translation might want to employ a strategy for checking, such as substituting a more familiar function in place of this $f(x)$, such as x^2. (A linear function may not be a good choice.) Plotting a few values on the curve $y = x^2 + 1$ would make it clear that this is a translation up by 1 unit. Answer choice C is a reflection about $y = x$, which students might choose if they mistakenly incorporate ideas about inverse functions. This question doesn't ask students to solve a problem or come up with a specific solution; it only asks them to think through principles of graphing and functions to arrive at the correct graph.

Test Question Rationale	
Functions	■ Apply basic trigonometric ratios to solve right-triangle problems ■ 28–32 score range

2. A distress call from a camper is received by 2 ranger stations. Station #1 is 10 miles due west from Station #2. The rangers determine that the camper is located as shown in the diagram below. How many miles is the camper from Station #2 ?



F. $\dfrac{10}{\sin 43°}$

G. $\dfrac{10}{\cos 43°}$

H. $10 \sin 43°$

J. $10 \cos 43°$

***K.** $10 \tan 43°$

Question 2 asks students to apply their knowledge of trigonometry to a real-world situation. One of the practical uses of trigonometry is to find angles and distances that are hard to measure directly. Here, a student who knows basic trigonometric relations in a right triangle can note that the sides of the triangle involved in the problem are the side opposite the 43° angle and the side adjacent to the 43° angle. The tangent function relates the lengths of these sides to the angle measure as follows:

$$\tan 43° = \frac{d}{10}$$

where d is the distance from the camper to Station #2. Solving this for d gives answer choice K. Other answers would be a result of misunderstanding or misapplying trigonometric functions.

While that is a good solution, there are many other ways to approach the problem. One of the more useful observations is that this is almost a 45°-45°-90°-triangle, so the distance from the camper to Station #2 should be about 10 miles, and it should really be a little less than 10 miles because 43° is slightly less than 45°. Approximating sin 43° and cos 43° as about 0.7 puts answer choices F and G at about 14, answer choices H and J at about 7, and answer choice K at about 10, making K the most reasonable choice. Even if students don't feel confident about using sine and cosine, using estimation skills gives them a way to feel confident about the solution.

The Assessment-Instruction Link

Why Is It Important to Link Assessment With Instruction?

Assessment provides feedback to the learner and the teacher. It bridges the gap between expectations and reality. Assessment can gauge the learners' readiness to extend their knowledge in a given area, measure knowledge gains, identify needs, and determine the learners' ability to transfer what was learned to a new setting.

When teachers use assessment tools to gather information about their students, then modify instruction accordingly, the assessment process becomes an integral part of teaching and learning. Using assessment to inform instruction can help teachers create a successful learning environment.

Students can use assessment as a tool to help them revise and rethink their work, to help integrate prior knowledge with new learning, and to apply their knowledge to new situations. Connecting assessment to classroom instruction can help both teachers and students take charge of thinking and learning.

As teachers review student performances on various measures, they can reexamine how to help students learn. As Peter Airasian, the author of *Classroom Assessment,* says, "Assessment is not an end in itself, but a means to another end, namely, good decision making" (p. 19). Linking assessment and instruction prompts both teachers and students to take on new roles and responsibilities. Through reflecting together on their learning, students and teachers can reevaluate their goals and embark on a process of continuous growth.

"*Every* objective, *every* lesson plan, *every* classroom activity, and *every* assessment method should focus on helping students achieve those [significant] outcomes that will help students both in the classroom and beyond."

— Kay Burke, editor of *Authentic Assessment: A Collection*

Are Your Students Developing the Necessary Skills?

Many high schools monitor the effectiveness of their educational program by tracking the success of their graduates after they leave high school. Some of the criteria by which schools measure success are the number of graduates who enroll in postsecondary institutions, the courses into which those students are placed, and the attrition rate of those students.

Because many colleges use ACT scores as one piece of information in making decisions about admissions and course placement, high schools can use students' ACT scores as they review their schools' performance. It is important to tie all the assessment information you gather to the goals of your mathematics program and to discuss how these goals are aligned with information about postsecondary institutions. With an ever-increasing number of high school graduates entering college, it becomes the school's responsibility to ensure that its graduates have mastered the prerequisite skills necessary for success in entry-level courses. ACT's Educational Planning and Assessment System (EPAS), of which EXPLORE, PLAN, and the ACT are each a part, can help provide information about students' level of knowledge and skills that can be used to guide students' secondary school learning experiences.

EXPLORE and PLAN are developmentally and conceptually linked to the ACT and thus provide a coherent framework for students and counselors and a consistent skills focus for teachers from Grades 8 through 12.

As students and others review test scores from EXPLORE, PLAN, and the ACT, they should be aware that ACT's data clearly reveal that students' ACT test scores are directly related to preparation for college. Students who take rigorous high school courses, which ACT has defined as core college preparatory courses, achieve much higher test scores than students who do not. ACT has defined core college preparatory course work as four or more years of English, and three or more years each of mathematics, social studies, and natural science.

ACT works with colleges to help them develop guidelines that place students in courses that are appropriate for their level of achievement as measured by the ACT. In doing this work, ACT has gathered course grade and test score data from a large number of first-year students across a wide range of postsecondary institutions. These data provide an overall measure of what it takes to be successful in a standard first-year college course. Data from 98 institutions and over 90,000 students were used to establish the ACT College Readiness Benchmark Scores, which are median course placement scores achieved on the ACT that are directly reflective of student success in a college course.

Success is defined as a 50 percent chance that a student will earn a grade of B or better. The courses are the ones most commonly taken by first-year students in the areas of English, mathematics, social studies, and science, namely English Composition, College Algebra, an entry-level College Social Studies/Humanities course, and College Biology. The ACT scores established as the ACT College Readiness Benchmark Scores are 18 on the English Test, 22 on the Mathematics Test, 21 on the Reading Test, and 24 on the Science Test. The College Readiness Benchmark Scores were based upon a sample of postsecondary institutions from across the United States. The data from these institutions were weighted to reflect postsecondary institutions nationally. The Benchmark Scores are median course placement values for these institutions and as such represent a *typical* set of expectations.

College Readiness Benchmark Scores have also been developed for EXPLORE and for PLAN, to indicate a student's probable readiness for college-level work, in the same courses named above, by the time the student graduates from high school. The EXPLORE and PLAN College Readiness Benchmark Scores were developed using records of students who had taken EXPLORE, PLAN, and the ACT (four years of matched data). Using either EXPLORE subject-area scores or PLAN subject-area scores, we estimated the conditional probabilities associated with meeting or exceeding the corresponding ACT Benchmark Score. Thus, each EXPLORE (1–25) or PLAN (1–32) score was associated with an estimated probability of meeting or exceeding the relevant ACT Benchmark Score. We then identified the EXPLORE and PLAN scores, at Grades 8, 9, 10, and 11, that came the closest to a 0.5 probability of meeting or exceeding the ACT Benchmark Score, by subject area. These scores were selected as the EXPLORE and PLAN Benchmark Scores.

All the Benchmark Scores are given in Table 4. Note that, for example, the first row of the table should be read as follows: An eighth-grade student who scores 13, or a ninth-grade student who scores 14, on the EXPLORE English Test has a 50 percent probability of scoring 18 on the ACT English Test; and a tenth-grade student who scores 15, or an eleventh-grade student who scores 17, on the PLAN English Test has a 50 percent probability of scoring 18 on the ACT English Test.

Table 4: **College Readiness Benchmark Scores**

	EXPLORE Test Score		**PLAN Test Score**		**ACT**
Subject Test	**Grade 8**	**Grade 9**	**Grade 10**	**Grade 11**	**Test Score**
English	13	14	15	17	18
Mathematics	17	18	19	21	22
Reading	15	16	17	19	21
Science	20	20	21	23	24

Using Assessment Information to Help Support Low-Scoring Students

Students who receive a Composite score of 16 or below on the ACT will most likely require additional guidance and support from their teachers and family in order to meet their post-high school goals, particularly if one of those goals is to attend a four-year college or university.

College admission policies vary widely in their level of selectivity. Students who score at or below 16 on the ACT might best be served by exploring those institutions that have an open or liberal admission policy. ACT Composite scores typically required by colleges having varying levels of selectivity are shown in Table 5. This information provides only general guidelines. There is considerable overlap among admission categories, and colleges often make exceptions to their stated admission policies.

A student's score on each content-area test on the ACT should also be reviewed with respect to his or her future goals. For example, a student who wishes to become an engineer will need a solid mathematics background. A high Mathematics Test score can be used as evidence that the goal is realistic. A low score (or subscore) suggests the student should consider ways of improving his or her mathematics skills through additional course work and/or additional assistance in the area.

What Are Some Factors That Affect Student Performance?

Many factors affect student achievement. Diane Ravitch, a research professor at New York University, has identified several positive factors in her book *The*

Table 5: **The Link Between ACT Composite Scores and College Admission Policies**

Admission Policy	Typical Class Rank of Admitted Students	Typical ACT Composite Scores of Admitted Students
Highly Selective	Majority of accepted freshmen in top 10% of high school graduating class	25–30
Selective	Majority of accepted freshmen in top 25% of high school graduating class	21–26
Traditional	Majority of accepted freshmen in top 50% of high school graduating class	18–24
Liberal	Some of accepted freshmen from lower half of high school graduating class	17–22
Open	All high school graduates accepted to limit of capacity	16–21

Schools We Deserve: Reflections on the Educational Crisis of Our Time (1985, pp. 276 and 294). These factors, which were common to those schools that were considered effective in teaching students, include

- a principal who has a clearly articulated vision for the school, and the leadership skills to empower teachers to work toward that vision;
- a strong, clearly thought-out curriculum in which knowledge gained in one grade is built upon in the next;
- dedicated educators working in their field of expertise;
- school-wide commitment to learning, to becoming a "community of learners";
- a blend of students from diverse backgrounds;
- "high expectations for all" students; and
- systematic monitoring of student progress through an assessment system.

There are also factors that have a negative impact on student achievement. For example, some students "may not know about, know how, or feel entitled to take academic advantage of certain opportunities, like college preparatory courses, college entrance exams, and extracurricular learning opportunities" (Goodwin, 2000, p. 3).

All students need to be motivated to perform well academically, and they need informed guidance in sorting out their educational/career aspirations. Teachers who challenge their students by providing a curriculum that is rigorous and relevant to their world and needs (Brewer, Rees, & Argys, 1995; Gay, 2000), and who have a degree and certification in the area in which they teach (Ingersoll, 1998) and ample opportunities to collaborate with their peers (McCollum, 2000), are more likely to engender students' success in school.

Making the Invisible Visible

Using assessment information, such as ACT's Educational Planning and Assessment System (EPAS), can help bring into view factors that may affect—either positively or negatively—student performance. Reviewing and interpreting assessment information can encourage conversations between parents and teachers about what is best for students. Using data is one way of making the assumptions you have about your students and school, or the needs of students, visible.

Collecting assessment information in a systematic way can help teachers in various ways. It can help teachers see more clearly what is happening in their classrooms, provide evidence that the method of teaching they're using really works, and determine what is most important to do next. As teachers become active teacher-researchers, they can gain a sense of control and efficacy that contributes to their sense of accomplishment about what they do each day.

There are many different types of assessment information that a school or school district can collect. Some types yield quantitative data (performance described in numerical terms), others qualitative data (performance described in nonnumerical terms, such as text, audio, video, or photographs, etc.). All types, when properly analyzed, can yield useful insights into student learning. For example, schools and teachers can collect information from

- standardized tests (norm- or criterion-referenced tests);
- performance assessments (such as portfolios, projects, artifacts, presentations);
- peer assessments;
- progress reports (qualitative, quantitative, or both) on student skills and outcomes;
- self-reports, logs, journals; and
- rubrics and rating scales.

Reviewing student learning information in the context of demographic data may also provide insight and information about specific groups of students, like low-scoring students. Schools therefore would benefit by collecting data about

- enrollment, mobility, and housing trends;

- staff and student attendance rates and tardiness rates;
- dropout, retention, and graduation rates;
- gender, race, ethnicity, and health;
- percent of free/reduced lunch and/or public assistance;
- level of language proficiency;
- staff/student ratios;
- number of courses taught by teachers outside their endorsed content area;
- retirement projections and turnover rates; and
- teaching and student awards.

What Does It Mean to Be a Low-Scoring Student?

Low-achieving students tend to be those students who score low on standardized tests. Students who slip behind are the likeliest to drop out and least likely to overcome social and personal disadvantages.

According to Judson Hixson, a researcher at the North Central Regional Educational Laboratory (NCREL), students who are at risk should be considered in a new light:

> Students are placed “at risk” when they experience a significant mismatch between their circumstances and needs, and the capacity or willingness of the school to accept, accommodate, and respond to them in a manner that supports and enables their maximum social, emotional, and intellectual growth and development.
>
> As the degree of mismatch increases, so does the likelihood that they will fail to either complete their elementary and secondary education, or more importantly, to benefit from it in a manner that ensures they have the knowledge, skills, and dispositions necessary to be successful in the next stage of their lives—that is, to successfully pursue post-secondary education, training, or meaningful employment and to participate in, and contribute to, the social, economic, and political life of their community and society as a whole.
>
> The focus of our efforts, therefore, should be on enhancing our institutional and professional capacity and responsiveness, rather than categorizing and penalizing students for simply being who they are. (Hixson, 1993, p. 2)

Hixson’s views reveal the necessity of looking at all the variables that could affect students’ performance, not just focusing on the students themselves.

Low-achieving students may demonstrate some of the following characteristics:

- difficulty with the volume of work to be completed;
- low reading and writing skills;
- low motivation;
- low self-esteem;
- poor study habits;
- lack of concentration;
- reluctance to ask for help with tasks/assignments; and
- test anxiety.

Many of these characteristics are interconnected. A low-scoring student cannot do the volume of work a successful student can do if it takes a much longer time to decipher text passages because of low reading skills. There is also the issue of intrinsic motivation in that students have little desire to keep trying to succeed if they habitually do not experience success.

But again, we must not focus only on the students themselves, but also consider other variables that could affect their academic performance, such as

- job or home responsibilities that take time away from school responsibilities;
- parental attitude toward and involvement in students’ school success;
- students’ relationships with their peers;
- lack of opportunities to engage in complex problems that are meaningful to students; and
- lack of adequate support and resources.

For example, some students who score low on tests are never introduced to a curriculum that challenges them or that addresses their particular needs: “Much of the student stratification within academic courses reflects the social and economic stratification of society. Schools using tracking systems or other methods that ultimately place low-income and marginal students in lower-level academic courses are not adequately preparing them to plan for postsecondary education, succeed in college, and prepare for lifelong learning” (Noeth & Wimberly, 2002, p. 18).

As Barbara Means and Michael Knapp have suggested, many schools need to reconstruct their curricula, employing instructional strategies that help students to understand how experts think through problems or tasks, to discover multiple ways to solve a problem, to complete complex tasks by receiving support (e.g., cues, modifications), and to engage actively in classroom discussions (1991).

Many individuals and organizations are interested in helping students succeed in the classroom and in the future. For example, the Network for Equity in Student Achievement (NESA), a group of large urban school systems, and the Minority Student Achievement Network (MSAN), a group of school districts in diverse suburban areas and small cities, are organizations that are dedicated to initiating strategies that will close the achievement gap among groups of students. Many schools and districts have found participation in such consortia to be helpful.

According to Michael Sadowski, editor of the *Harvard Education Letter,* administrators and teachers who are frustrated by persistent achievement gaps within their school districts “have started to look for answers within the walls of their own schools. They’re studying school records, disaggregating test score and grade data, interviewing students and teachers, administrating questionnaires—essentially becoming researchers—to identify exactly where problems exist and to design solutions” (Sadowski, 2001, p. 1).

A student may get a low score on a standardized test for any of a number of reasons. To reduce the probability of that outcome, the following pages provide some suggestions about what educators and students can do before students’ achievement is assessed on standardized tests like the ACT.

What Can Educators and Students Do Before Students Take the ACT?

Integrate assessment and instruction. Because the ACT is curriculum based, the most important prerequisite for optimum performance on the test is a sound, comprehensive educational program. This “preparation” begins long before any test date. Judith Langer, the director of the National Research Center on English Learning and Achievement, conducted a five-year study that compared the English programs of typical schools to those that get outstanding results. Schools with economically disadvantaged and diverse student populations in California, Florida, New York, and Texas predominated the study. Langer’s study revealed that in higher performing schools “test preparation has been integrated into the class time, as part of the ongoing English language arts learning goals.” This means that teachers discuss the demands of high-stakes tests and how they “relate to district and state standards and expectations as well as to their curriculum” (Langer, Close, Angelis, & Preller, 2000, p. 6).

Emphasize core courses. ACT research conducted in urban schools both in 1998 and 1999 shows that urban school students can substantially improve their readiness for college by taking a tougher sequence of core academic courses in high school. Urban students taking a more rigorous sequence of courses in mathematics and science and finding success in those courses score at or above national averages on the ACT. Regardless of gender, ethnicity, or family income, those students who elect to take four or more years of rigorous English courses and three or more years of rigorous course work in mathematics, science, and social studies earn higher ACT scores and are more successful in college than those who have not taken those courses (ACT & Council of Great City Schools, 1999). Subsequent research has substantiated these findings and confirmed the value of rigor in the core courses (ACT, 2004; ACT & The Education Trust, 2004).

Teach test-taking strategies. Students may be helped by being taught specific test-taking strategies, such as the following:

- Learn to pace yourself.
- Know the directions and understand the answer sheet.

- Read carefully and thoroughly.
- Answer easier questions first; skip harder questions and return to them later.
- Review answers and check work, if time allows.
- Mark the answer sheet quickly and neatly; avoid erasure marks on the answer sheet.
- Answer every question (you are not penalized for guessing on the ACT).
- Become familiar with test administration procedures.
- Read all the answer choices before you decide which is the correct answer.

Students are more likely to perform at their best on a test if they are comfortable with the test format, know appropriate test-taking strategies, and are aware of the test administration procedures. Test preparation activities that help students perform better in the short term will be helpful to those students who have little experience taking standardized tests or who are unfamiliar with the format of the ACT.

Search out other sources of help. School personnel in urban or high-poverty middle schools can investigate programs such as GEAR-UP, which "provides federal funds for schools to prepare low-income middle school students for high school and college preparation through multiple school reform efforts. School districts, colleges, community organizations, and businesses often form partnerships to provide teachers with enhanced professional development opportunities to ensure they have the necessary tools and strategies to teach middle school and high school effectively" (Noeth & Wimberly, 2002, p. 18).

What Do the ACT Mathematics Test Results Indicate About Low-Scoring Students?

Students who score below 16 on the ACT Mathematics Test are likely to have some or all of the knowledge and skills described in the ACT Mathematics College Readiness Standards for the 13–15 range. In fact, they may well have some of the skills listed in the 16–19 range. However, these students need to become more consistent in demonstrating these skills in a variety of contexts or situations.

The EPAS Mathematics College Readiness Standards indicate that students who score below 16 tend to be able to

- Perform one-operation computation with whole numbers and decimals
- Solve problems in one or two steps using whole numbers
- Perform common conversions (e.g., inches to feet or hours to minutes)
- Calculate the average of a list of positive whole numbers
- Perform a single computation using information from a table or chart
- Recognize equivalent fractions and fractions in lowest terms
- Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$)
- Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals
- Identify the location of a point with a positive coordinate on the number line
- Estimate or calculate the length of a line segment based on other lengths given on a geometric figure

In sum, these students typically show skill working primarily with whole numbers and decimals, whether they are looking at data, solving equations, or dealing with measuring geometric figures. These students will likely benefit from encouragement in performing calculations and solving equations involving rational numbers; in extending their knowledge of graphing to the coordinate plane; in becoming more comfortable with the basic concepts of probability, statistics, and data analysis through real-world problems; and in extending measurement concepts to include perimeter and area for a variety of geometric figures.

What Does Research Say About How Mathematics Instruction Should Be Conducted?

Research suggests that learning is maximized when students take a demanding core curriculum and engage in rigorous learning activities. The core curriculum must be embedded in a learning environment where students are motivated to work hard. To be motivated, students need to see the relevance of their schoolwork (LaPoint, Jordan, McPartland, & Towns, 1996). Research also suggests that framing learning and performance tasks within contexts that are familiar cultural experiences may improve students' cognitive functioning and consequently their achievement (Boykin & Bailey, 2000).

National Council of Teachers of Mathematics (NCTM) recommendations for change in mathematics education call for teachers to use a wide range of instructional strategies:

> A variety of instructional methods should be used in classrooms to cultivate students' abilities to investigate, make sense of, and construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition to traditional teacher demonstrations and teacher-led discussions, greater opportunities should be provided for small-group work, individual explorations, peer instruction, and whole-class discussions in which the teacher serves as a moderator. (NCTM, 1989, pp. 125, 128)

What Can Be Done to Help Students Understand the Mathematical Situations They Encounter?

Students need to see how mathematics crosses into other disciplines and is used in real life, not just as isolated pieces of information to be used only in mathematics class. Today's mathematics instruction should not be just about memorization, though some things like the basic facts of arithmetic need to be internalized to the point of automatic recall. The memorization-and-drill approach does not provide an in-depth understanding of those mathematical concepts required to build a strong foundation in math, nor does the lecture method.

The "scaffolding" method, in which the teacher sets up interactive learning activities at increasingly challenging levels as students progress toward mastery of a concept, can be used as an alternative to drills and lecture. The learning activities can be organized so that the students are interacting with the teacher or with their peers. The teacher needs to monitor group discussions and guide the students as they use strategies of questioning, summarizing, clarifying, and predicting to achieve comprehension of the major concept(s) being taught. For example, a teacher could increase the level of difficulty in the Algebraic Equations instructional activity on pages 58–61 by using a larger coefficient of the variable so that students would have to think more about how to sort and group the equations. Another idea using this same activity would be to have those students who have internalized how to solve algebraic equations work with others who still need assistance, by playing a game of "concentration." The teacher could make a set of cards with simple one-step equations and their solutions and have the students pair a one-step equation with the correct solution as a "match." The level of difficulty for the game could be increased by making another set of cards that include a variety of one- and two-step equations. Each student takes a turn selecting cards until he/she gets a matched pair (one-step equations) or matched group (two-step equations). Play would continue until all of the cards were selected, and a player's standing within the group could be calculated by assigning points for each "match." Matched groups that involve two steps for their solution could be worth more points.

Mathematics study should include investigation of patterns in numbers, shapes, data, probability, and growth/decay, along with practice in communicating mathematically. Students should also be encouraged to guess:

> Part of developing student confidence is the realization that guessing plays an important role in learning mathematics. Even so, students must learn that the guesses must be tempered by validation and attempts to structure an explanation as to why the guess is an appropriate response to the situation at hand. Developing the ability to conjecture, test, revise, and reconjecture is an important step toward thinking mathematically. (Association for Supervision and Curriculum Development [ASCD], 1999, p. 9)

Today, mathematics requires students to move beyond memorization and drill and to develop skills that include problem solving, reasoning, representation, and communication. Students need to be able to visualize and make connections between the various branches of mathematics and between mathematics and other content areas. Students might need to solve a probability problem geometrically by using coordinates from a Cartesian plane, or to use their knowledge of number concepts, properties, and theorems (such as exponentiation, root extraction, and the Pythagorean theorem) along with their algebraic manipulation skills to solve a measurement problem involving perimeter, area, or volume of a composite geometric figure. Teachers need to guide their students through activities such as small-group work, individual explorations, use of technology, peer instruction, and math logs/journals so that the students have the opportunity to develop reasoning and problem-solving skills—skills they'll need to use when confronted with purely mathematical problems and in real-world situations with connections to other content areas.

Teachers also need to help students understand difficult or abstract concepts by using real-life examples to make the concepts more concrete. For example, the order of operations to solve two-step algebra equations could be reinforced using shoes and socks. When students get dressed they put on socks first, then shoes; but when they want to undress, they do the reverse: they remove instead of put on, removing their shoes first and socks second. In analyzing the equation $2x + 3 = 10$, the order of operations is to multiply x by 2 and then add 3. To solve the equation, the inverse operations are used in the reverse order: subtract 3 and then divide by 2.

Analyze the equation	→	get dressed
Multiply x by 2	→	put on socks
Add 3	→	put on shoes
Solve the equation	→	get undressed
Subtract 3	→	take off shoes
Divide by 2	→	take off socks

By connecting the abstract to the concrete as in the shoes and socks example, the teacher can reinforce mathematical concepts by relating them to situations in everyday life.

What Knowledge and Skills Are Low-Scoring Students Ready to Learn?

For students who score below 16 on the ACT Mathematics Test, their target achievement outcomes could be the College Readiness Standards listed in the 16–19 range:

- Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent
- Solve some routine two-step arithmetic problems
- Calculate the average of a list of numbers
- Calculate the average, given the number of data values and the sum of the data values
- Read tables and graphs
- Perform computations on data from tables and graphs
- Use the relationship between the probability of an event and the probability of its complement
- Recognize one-digit factors of a number
- Identify a digit's place value
- Substitute whole numbers for unknown quantities to evaluate expressions
- Solve one-step equations having integer or decimal answers
- Combine like terms (e.g., $2x + 5x$)
- Locate points on the number line and in the first quadrant
- Exhibit some knowledge of the angles associated with parallel lines
- Compute the perimeter of polygons when all side lengths are given
- Compute the area of rectangles when whole number dimensions are given

By no means should these be seen as limiting or exclusive goals. As stated earlier, it is important to use multiple sources of information to make instructional decisions and to recognize that individual students learn at different rates and in different sequences. What's important is to get students communicating mathematically.

What Strategies/Materials Can Teachers Use in Their Classrooms?

According to Bryan Goodwin, senior program associate at the Mid-continent Research Education Laboratory (McREL), "it is important to note that improving the performance of disenfranchised students does not mean ignoring other students. Indeed, many of the changes advocated—such as making curricula more rigorous and creating smaller school units—will benefit all students" (Goodwin, 2000, p. 6). Means and Knapp (1991) express a similar view:

> A fundamental assumption underlying much of the curriculum in America's schools is that certain skills are "basic" and must be mastered before students receive instruction on more "advanced" skills, such as reading comprehension, written composition, and mathematical reasoning. . . . Research from cognitive science questions this assumption and leads to a quite different view of children's learning and appropriate instruction. By discarding assumptions about skill hierarchies and attempting to understand children's competencies as constructed and evolving both inside and outside of school, researchers are developing models of intervention that start with what children know and provide access to explicit models of thinking in areas that traditionally have been termed "advanced" or "higher order." (p. 1)

Pages 48–61 exemplify the kind of teacher-developed activities that could be used in a classroom for all students, not just those who have scored low on a standardized assessment like the ACT. The first activity, which has two parts, has students perform routine one-operation computation with whole numbers and decimals as they look for patterns. The students are then asked to draw a conclusion about or describe the pattern(s) they discovered. Two Student Activity Sheets and two Rating Scale Checklists are included that could be used by the teacher to assess student learning. Also included is a Math Journal Checklist that could be used either by the teacher or student to evaluate entries in a math journal. The second activity focuses on solving algebraic equations. Students are asked to identify and explain/justify the proper sequence of steps leading to the solution for each equation included in their group's assigned envelope. A simple Algebraic Scoring Rubric is included, along with a Group Questionnaire and suggestions for other types of assessments that could be used to evaluate the students' learning. Both sets of activities provide suggestions for related investigations that would enable the teacher to extend and build on the original ideas in order to reinforce and strengthen student learning.

How Are the Activities Organized?

For each activity, the primary strand or strands are displayed across the top of the page. (The strand name "Probability, Statistics, & Data Analysis" is abbreviated "Data Analysis.") Next is a box that contains *Guiding Principles*—statements about instruction, assessment, thinking skills, student learning, and other educationally relevant topics. (The bibliography at the end of this guide includes the source for each statement referenced.) Following the *Guiding Principles* box is the title of the activity, followed by the relevant *College Readiness Standards,* then the *Description of the Instructional Activity.* Each description is followed by *Suggestions for Assessment,* the applicable *Ideas for Progress,* and *Suggested Strategies/Activities.*

- The *College Readiness Standards* section lists the skill statements tied directly to the strand or strands that will be focused on in the activity.
- The *Description of the Instructional Activity* section provides suggestions for engaging students in the activity, and, where applicable, gives a list of related topics or tasks. The activities address a range of objectives and modes of instruction, but they emphasize providing students with experiences that focus on reasoning and making connections, use community resources and real-life learning techniques, and encourage students to ask questions—questions leading to analysis, reflection, and further study and to individual construction of meanings and interpretations.
- The *Suggestions for Assessment* section offers ideas for documenting and recording students' learning during an instructional activity.
- The *Ideas for Progress* section provides statements that suggest learning experiences that are connected to the *Suggested Strategies/Activities.*
- The *Suggested Strategies/Activities* section provides an elaboration on the central activity or ideas for investigating related topics or issues. The suggestions are connected to one or more ideas for progress.

These teacher-developed activities provide suggestions, not prescriptions. You are the best judge of what is necessary and relevant for your students. Therefore, we encourage you to review the activities, modifying and using those suggestions that apply, and disregarding those that are not appropriate for your students. As you select, modify, and revise the activities, you can be guided by the statements that appear in the *Guiding Principles* box at the beginning of each activity.

Guiding Principles

- "The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase." (National Council of Teachers of Mathematics [NCTM], 2000, p. 4)
- "Students learn to communicate in a variety of ways by actively relating physical materials, pictures, and diagrams to mathematical ideas, by reflecting upon and clarifying their own thinking, by relating everyday language to mathematical ideas and symbols, and by discussing mathematical ideas with peers." (Zemelman, Daniels, & Hyde, 1993, p. 75)
- "To ensure deep, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption." (NCTM, 2000, p. 23)

Recognizing Patterns

College Readiness Standards

- Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor
- Perform one-operation computation with whole numbers and decimals
- Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent

Description of the Instructional Activity

Students need opportunities to explore basic operations and applications by performing a series of activities that lead them to recognize patterns. Many such activities are available to teachers. Here are two examples of repetitive, pattern-developing activities that could be used with students who struggle with the basic concepts (adapted from materials created by Marilyn Stor, educational consultant for Alameda High School, Jefferson County, Colorado, 2001).

Activity A: Fraction Patterns

Give each student a Looking for Patterns worksheet (see pages 51–52). Have students complete the first column by converting the fraction to a decimal. Give them an opportunity to experience some success by allowing them to complete the fraction conversions they already know. Each student can then pair with another student to finish converting the remaining fractions to decimals. Next have them convert each decimal to a percent. As the students fill in the decimal columns they should begin to discover patterns. For example, in the fraction group with a denominator of 9, the fraction 1/9 becomes a decimal of .111..., the fraction 2/9 becomes a decimal of .222..., and students can start to see a pattern that 3/9 would be .333.... Again, it is the patterns that you are trying to help them recognize. Give them ample time to complete the entire sheet. Ideally, they should complete this activity without using a calculator.

Activity B: Rule of 70

This activity also allows students to develop a mathematical concept by recognizing patterns. The activity relates to compound interest in a very hands-on manner by having students measure out a length of paper tape to represent money that is saved or invested, allowing them to see how money grows over a period of time. Depending upon the students' prior knowledge and experience, pre-activities dealing with the relationship between a meter and a centimeter and how to change percents to decimals on a calculator might be needed.

Each small group (3–4 students) should be provided with a cash register tape (approximately 2.5 meters long), a meter stick, tape, scissors, and a calculator. Each student should receive a worksheet to record their group's results (see sample "Double Your Money" Student Activity Sheet on page 53). Have each group cut off a piece of their cash register tape 1 meter long. This piece represents $100. Assign an interest rate to each group (e.g., 5%, 6%, 7%, 8%, 9%, 10%, 12%, 14%).

Have the groups use their assigned percentage rate to calculate the amount of money earned in interest during the first year of savings and record it on each student's worksheet within the group. Each group should then measure and cut a second piece of cash register tape that appropriately represents the money earned during the first year of savings and tape it end-to-end to the $100 paper tape. Have the students record their tape's new length, which represents the dollar value, into their table. Have the groups repeat these steps using their assigned interest rate until each group has doubled their $100.

After all the groups have completed the activity and doubled their money, collect the group data to display on an overhead transparency (see page 54). Ask the students to describe the pattern(s) they see in the table. Recognizing an inverse relationship (column two decreases as column one increases) is a good conclusion. Allow the students to discover that the Rule of 70 means that your money is doubled when the product of the number of years times the interest rate is 70 (e.g., the product of the R and T from the continuous compounding formula $Q = Q_0e^{RT}$ is approximately 0.70 when $Q = 2Q_0$).

Suggestions for Assessment

Anecdotal Notes—Students could be observed as they complete the activity worksheets. The teacher could note whether students are actively participating in the mathematical calculations and accurately modeling their results.

Math Journal Checklist—Students could be asked to keep a journal to document the various types of problems they encounter. Among the entries students could record in their journal are sample problems with written explanations of the process(es) used to solve them or to draw an appropriate conclusion. A Math Journal Checklist could be used to evaluate students' entries in the Math Journal. The teacher and/or students could generate the criteria for the checklist. The checklist may be developed in various ways: it may cover a wide range of skills and understandings, it may cover a specific set of skills and understanding, or it may change as the unit unfolds to reflect new skills and understandings being learned. The checklist on page 55, for example, assesses such broad skills as students' mathematical understandings, use of mathematical language, and mathematical reasoning or argumentation. It also assesses the students' abilities to make inferences and generalizations, to ask thoughtful questions, and to reflect on their own thinking processes. The math journal could continue throughout the entire unit.

Teachers could use the checklist in several ways. For example, teachers should be sure students understand that the checklist is to be used during the entire unit and that all the criteria on the checklist should be illustrated somewhere in the journal, but certainly all would not be used in each entry. Another approach might be for a teacher to specify certain checklist criteria to be used in certain entries, or for a teacher to ask students to choose one or two criteria from the checklist to focus on as they write each journal entry. The checklist could be used either by the teacher or students to evaluate entries in the math journal.

Rating Scale Checklist—A qualitative Rating Scale Checklist, such as the ones on pages 56–57, could be used to assess students' mathematical skills and understandings, how well they communicated their ideas, and the quality and accuracy of their work.

Ideas for Progress

- Apply elementary number concepts, including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring
- Create expressions that model mathematical situations using combinations of symbols and numbers

Suggested Strategies/Activities

Students could be given lists of numbers that form a pattern (e.g., 100, 97, 94, 91, ...), pictorial representations of sequences (e.g., triangular numbers), or a situation that forms a sequence (e.g., the cost of purchasing an increasing number of gallons of gas for your car). The students could then be asked to generate the next several terms and describe the rule for the sequence using words and/or algebraic notation. A well-known sequence that is easy to describe in words, but not algebraically, is the Fibonacci sequence. Students could generate terms, describe the rule, and research and report about the Fibonacci sequence and its occurrence in nature (e.g., patterns in pinecones, flower petals, leaf arrangements).

Looking for Patterns Student Activity Sheet

Name: ______________________________ Period: ______________ Date: ______________

Scenario: You have recently acquired part ownership in a company. To determine the percent of profit you and your partners have made thus far, change the indicated fractions to decimals and %. When you find a pattern for a group such as the 1/9ths, describe it on the lines provided at the end of this worksheet. Circle the ones that you filled in from memory. Highlight the ones you filled in once you discovered a pattern. Attach your work.

Fraction	Decimal	Percent	Fraction	Decimal	Percent	Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{1}$			$\frac{1}{2}$			$\frac{1}{3}$			$\frac{1}{4}$		
$\frac{2}{1}$			$\frac{2}{2}$			$\frac{2}{3}$			$\frac{2}{4}$		
$\frac{3}{1}$			$\frac{3}{2}$			$\frac{3}{3}$			$\frac{3}{4}$		
$\frac{4}{1}$			$\frac{4}{2}$			$\frac{4}{3}$			$\frac{4}{4}$		
$\frac{5}{1}$			$\frac{5}{2}$			$\frac{5}{3}$			$\frac{5}{4}$		
$\frac{6}{1}$			$\frac{6}{2}$			$\frac{6}{3}$			$\frac{6}{4}$		
$\frac{7}{1}$			$\frac{7}{2}$			$\frac{7}{3}$			$\frac{7}{4}$		
$\frac{8}{1}$			$\frac{8}{2}$			$\frac{8}{3}$			$\frac{8}{4}$		

Fraction	Decimal	Percent	Fraction	Decimal	Percent	Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{5}$			$\frac{1}{6}$			$\frac{1}{7}$			$\frac{1}{8}$		
$\frac{2}{5}$			$\frac{2}{6}$			$\frac{2}{7}$			$\frac{2}{8}$		
$\frac{3}{5}$			$\frac{3}{6}$			$\frac{3}{7}$			$\frac{3}{8}$		
$\frac{4}{5}$			$\frac{4}{6}$			$\frac{4}{7}$			$\frac{4}{8}$		
$\frac{5}{5}$			$\frac{5}{6}$			$\frac{5}{7}$			$\frac{5}{8}$		
$\frac{6}{5}$			$\frac{6}{6}$			$\frac{6}{7}$			$\frac{6}{8}$		
$\frac{7}{5}$			$\frac{7}{6}$			$\frac{7}{7}$			$\frac{7}{8}$		
$\frac{8}{5}$			$\frac{8}{6}$			$\frac{8}{7}$			$\frac{8}{8}$		
$\frac{9}{5}$			$\frac{9}{6}$			$\frac{9}{7}$			$\frac{9}{8}$		

Looking for Patterns Student Activity Sheet *(continued)*

Fraction	Decimal	Percent	Fraction	Decimal	Percent	Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{9}$			$\frac{1}{10}$			$\frac{1}{11}$			$\frac{1}{12}$		
$\frac{2}{9}$			$\frac{2}{10}$			$\frac{2}{11}$			$\frac{2}{12}$		
$\frac{3}{9}$			$\frac{3}{10}$			$\frac{3}{11}$			$\frac{3}{12}$		
$\frac{4}{9}$			$\frac{4}{10}$			$\frac{4}{11}$			$\frac{4}{12}$		
$\frac{5}{9}$			$\frac{5}{10}$			$\frac{5}{11}$			$\frac{5}{12}$		
$\frac{6}{9}$			$\frac{6}{10}$			$\frac{6}{11}$			$\frac{6}{12}$		
$\frac{7}{9}$			$\frac{7}{10}$			$\frac{7}{11}$			$\frac{7}{12}$		
$\frac{8}{9}$			$\frac{8}{10}$			$\frac{8}{11}$			$\frac{8}{12}$		
$\frac{9}{9}$			$\frac{9}{10}$			$\frac{9}{11}$			$\frac{9}{12}$		
$\frac{10}{9}$			$\frac{10}{10}$			$\frac{10}{11}$			$\frac{10}{12}$		
$\frac{11}{9}$			$\frac{11}{10}$			$\frac{11}{11}$			$\frac{11}{12}$		
$\frac{12}{9}$			$\frac{12}{10}$			$\frac{12}{11}$			$\frac{12}{12}$		

Give a rule or describe the pattern you discovered for each fraction “family.”

“Double Your Money” Student Activity Sheet

Name: ______________________________ Period: ____________ Date: ____________

If you plan to invest some money, how long do you think it will take for your money to DOUBLE its value? ____________ What factor(s) do you think would influence this length of time?

____

Each group is going to make a model that displays how your money grows. Your group is going to invest $100 in an account that pays ______% per year and compounds once per year.

Step 1: Cut a paper tape that is 1 meter long to represent $100. How much money would each centimeter represent? ________

Step 2: Assume you are at the end of your first year of savings. Use your group's percentage rate to find the amount of money earned in interest during the first year. ________

- Measure and cut a piece of paper tape from your group's materials that appropriately represents the money earned during this first year of savings.
- Tape it end-to-end to the $100 paper tape.
- What is the new length? Record this information in the table.
- Repeat the process of calculating the interest, adding the appropriate length of paper tape to your model, and recording the information in the table until the $100 has doubled its value.

Length of paper tape and $Value$	**Year**	**Interest at ______%**	**New length of paper tape and $Value$**
1 m = $100	Beginning Year 0		
	Year 1		
	Year 2		
	Year 3		

Conclusion for Doubling Your Money: ______________________________

____

Group Data Table

Rate (%)	Time to double (in years)	Rate • Time	Rate + Time
5%			
6%			
7%			
8%			
9%			
10%			
12%			
14%			

Math Journal Checklist

Name: ______________________________ Period: ______________ Date: ______________

(The blanks at the end are for additional criteria generated by the teacher and/or students.)

Criteria	**Yes/No** (include dates that journal is reviewed)	**Comments**
Makes connections between mathematics and other disciplines or applied fields		
Generally uses precise mathematical language and notation		
Explains own thinking; may use mathematical models, drawings, facts, etc.		
Provides both complete and accurate mathematical information		
Uses logical reasoning to make conjectures, to construct arguments, and/or to validate and prove conclusions		
Analyzes arguments, recognizing fallacies or underlying assumptions		
Makes inferences and/or generalizations based on logic and probability		
Poses thoughtful questions		
Reflects on and assesses own thinking and learning		
Writes about mathematical materials read with understanding		
Analyzes new information gained		

Rating Scale Checklist

Name: ______________________ Period: __________ Date: __________

Directions: Note the degree of evidence the student has demonstrated for each criterion.

Activity A: Fraction Patterns Criteria and Scoring				
Quality of Work	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Worked diligently to convert fractions to decimals from memory				
■ Worked diligently with partner to finish fraction to decimal conversions				
■ Worked diligently with partner to convert each decimal to a percent				
Accuracy of Work	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Decimal columns filled in accurately				
■ Percent columns filled in accurately				
Mathematical Skills and Understandings	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Shows understanding of pattern repetition in decimal columns				
■ Shows understanding of pattern recognition when converting decimals to percents				
Organization of Information	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Has included rule or description for each fraction “family”				
■ Has included circles and highlights				
■ Has attached computation work				
Communication of Ideas	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Provides a well-reasoned explanation of the pattern				
■ Uses appropriate mathematical language to express ideas				
■ Refers to various aspects of the worksheet to explain own thinking				
Column Totals				

Total Score for Activity A ______

Rating Scale Checklist

Name: ______________________ Period: __________ Date: __________

Directions: Note the degree of evidence the student has demonstrated for each criterion.

Activity B: Rule of 70 Criteria and Scoring				
Quality of Work	**Exemplary Evidence** 4	**Much Evidence** 3	**Some Evidence** 2	**Little Evidence** 1
■ Performed individual assigned duty on the team				
■ Worked diligently with team to complete "Double Your Money" Student Activity Sheet				
Accuracy of Work	**Exemplary Evidence** 4	**Much Evidence** 3	**Some Evidence** 2	**Little Evidence** 1
■ Interest column filled in accurately				
■ New $Value$ column filled in accurately				
Mathematical Skills and Understandings	**Exemplary Evidence** 4	**Much Evidence** 3	**Some Evidence** 2	**Little Evidence** 1
■ Shows understanding of % as an interest rate				
■ Shows understanding of growth (amount of money earned in interest)				
■ Shows understanding of the relationship between the length of the paper tape and the $Value$				
Organization of Information	**Exemplary Evidence** 4	**Much Evidence** 3	**Some Evidence** 2	**Little Evidence** 1
■ Has completed the Student Activity Sheet with the results obtained by the group				
■ Has included conclusion(s) after discussion about the group data recorded on the overhead transparency				
Communication of Ideas	**Exemplary Evidence** 4	**Much Evidence** 3	**Some Evidence** 2	**Little Evidence** 1
■ Provides a well-reasoned explanation of the conclusion/pattern				
■ Uses appropriate mathematical language to express ideas				
■ Refers to various aspects of the worksheet to explain own thinking				
Column Totals				

Total Score for Activity B ______

Guiding Principles

- "All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding." (NCTM, 2000, p. 5)
- "Understanding develops as students construct new relationships among ideas, as they strengthen existing relationships among those ideas, and as they reorganize their ideas." (Secada, 1997, p. 8)
- "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students." (NCTM, 2000, p. 22)

Algebraic Equations

College Readiness Standards

- Combine like terms (e.g., $2x + 5x$)
- Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals
- Solve one-step equations having integer or decimal answers
- Solve routine first-degree equations

Description of the Instructional Activity

The activity is designed to give students guided practice in solving multistep equations, providing reinforcement of mathematical concepts through additional peer instruction time while working in a group setting. Depending upon the students' prior knowledge and experience, pre-activities that have students combine like terms and perform computations where integers and fractions are used as inverses might be needed before starting this activity.

On a sheet of paper, write out several algebraic first-degree equations and the step-by-step solution of each equation. Be sure to use a different variable for each equation.

Example 1: $3x = 3$
$x = 1$

Example 2: $4p + 7 = 47$
$4p = 40$
$p = 10$

Make a copy of the equations sheet for each group. Cut the equations and their step-by-step solution into strips, shuffle them, and place them in an envelope. Distribute an envelope to each group.

The task for students, working in their groups, is to sort the steps on the strips of paper into the proper sequence to solve the equation. The teacher's task is to circulate through the room and observe the interaction within the groups; check on each student's participation and understanding; and provide guidance and instruction as needed.

When the teacher has verified that the groups have performed the task of putting all of the steps in the proper sequence for each equation, the next task is for the students to write down each equation and its proper sequence of steps leading to the solution. To reinforce the process, have each student write an explanation or justification for each step (e.g., what operation or property was used to get from one step to the next). A list of the property names with an example for each property could be provided for each group to aid students with their explanations.

After all the groups have completed the activity, each student could be given a new set of equations to solve independently. (This activity is adapted from Rubenstein et al., *Integrated Mathematics 1,* 1995, p. 251)

Suggestions for Assessment

Group Questionnaire—As a group, the students could complete a questionnaire designed to encourage cooperative working relationships and to help them develop an effective plan of action for working with the equations (see sample questionnaire on page 60).

Anecdotal Notes—The teacher can observe the students, noting those who are and are not actively participating in the ordering of the steps for the solution to each equation.

Pencil-and-Paper Tasks—To evaluate each student's level of understanding, students could be given a set of equations as a quiz/test and asked to match the property or operation that would justify each step.

Rubric—A rubric could be used to assess student participation within a group, student performance of the key steps involved in the equation-sequencing-and-justification task, and demonstration of mastery of the process on a test (see rubric on page 61).

Ideas for Progress

- Evaluate algebraic expressions and solve multistep first-degree equations
- Use the inverse relationships for the four basic operations, exponentiation, and root extractions to determine unknown quantities
- Create and solve linear equations and inequalities that model real-world situations

Suggested Strategies/Activities

Students could redo the activity with the same algebraic equations or with several new algebraic equations where each equation uses the same variable.

Give each group several equations and have the students classify the equations as solvable with only addition/subtraction, solvable with only multiplication/division, or requiring both operations. After separating the equations into the three categories, the students could decide what operation(s) need(s) to be performed and then write and explain or justify each step leading to the correct solution.

The teacher could have each group create their own equations, identifying the order of the steps that lead to the correct solution. They could distribute their packet of equations to each of the other groups and challenge them to see which group could correctly order the steps in the shortest amount of time.

Students could also be given expressions or equations and asked to write a real-world problem that the expression or equation could be modeling. Teachers could extend the real-world concept even further by giving students equations or inequalities found in a break-even business situation that might include fixed costs and variable costs.

Group Questionnaire

Names/Group: ______________________________ Period: __________

This questionnaire is designed to be used by all the members of a group to guide their planning and development of an effective plan of action and to reflect on their problem-solving strategies and decision-making skills.

Directions: 1. As a group, carefully read each question.
2. Place a "✓" and a date in the scoring box that applies.
3. Discuss each question with the members of your group and write a group response.

Scoring		**Questions**	**Responses**
Yes	Not quite	Can you explain the tasks(s) and its key elements?	Task(s):
Yes	Not quite	Have you thought about and discussed possible plans for completing the task(s)?	Best Solution:
Yes	Not quite	Can you list steps for completing the task(s)?	Steps in process:
Yes	Not quite	Did you use an appropriate strategy for each step and sub-step?	Strategies used:
Yes	Not quite	Explain how this task is similar to other tasks you have encountered.	Explanation:
Yes	Not quite	Did the strategies your group chose and the decisions your group made work?	Reasons:

Algebraic Equations Rubric

Name: ______________________________ Period: ____________ Date: ____________

Directions: Note the degree of evidence the student has demonstrated for each criterion.

Criteria	4 Exemplary	3 Accomplished	2 Developing	1 Beginning	Score
Working in a group 10%	■ Takes the initiative to get the group working immediately ■ Performs all the duties assigned ■ On task at all times	■ Starts working when prompted by another student ■ Performs nearly all the duties assigned ■ Stays on task most of the time	■ Starts working when prompted by the teacher ■ Performs few of the duties assigned ■ Needs reminders to stay on task	■ Relies on others to do the task ■ Does not perform the duties assigned ■ Does not stay on task	
Determining steps for equations 25%	■ Every equation is grouped by variable ■ Every step is in the proper order	■ Every equation is grouped by variable ■ Only one step is out of order	■ The equations are grouped by variable ■ Two or more steps are out of order	■ The equations are not grouped by variable ■ The steps are not in the proper order	
Explaining steps for equations 25%	■ Every equation, with all of the steps in proper order, is on the student's paper ■ All of the explanations are correct	■ Every equation, with all of the steps in proper order, is on the student's paper ■ Although all explanations are present, some explanations do not justify the step	■ Most of the equations are on the student's paper ■ Not all of the explanations are present	■ Some of the equations are on the student's paper ■ None of the steps are explained or justified	
Solving new algebraic equations 40%	■ Every new equation is solved ■ Every step is shown	■ Every new equation is solved ■ Almost every step is shown	■ Some of the new equations are solved ■ Some of the work is shown	■ The equations are not solved ■ No work is shown	

Total Score ______

Instructional Activities for ACT Mathematics

Why Are Additional Instructional Activities Included?

The set of instructional activities that begins on page 64 was developed to illustrate the link between classroom-based activities and the skills and understandings embedded in the ACT Mathematics Test questions. The activities are provided as examples of how classroom instruction and assessment, linked with an emphasis on reasoning, can help students practice skills and understandings they will need in the classroom and in their lives beyond the classroom. It is these skills and understandings that are represented on the ACT Mathematics Test.

A variety of thought-provoking activities, such as applying mathematics, making connections within mathematics and to other areas, small- and large-group discussions, and both independent and collaborative activities, are included to help students develop and refine their skills in many types of situations.

The instructional activities that follow have the same organizational structure as the ones in the previous section. *Like the other activities, these activities were not developed to be a ready-to-use set of instructional strategies.* ACT's main purpose is to illustrate how the skills and understandings embedded in the ACT Mathematics Test questions can be incorporated into classroom activities.

For the purpose of this part of the guide, we have tried to paint a picture of the ways in which the activities could work in the classroom. We left room for you to envision how the activities might best work for you and your students. We recognize that as you determine how best to serve your students, you take into consideration your teaching style as well as the academic needs of your students; state, district, and school standards; and available curricular materials.

The instructional activities are not intended to help drill students in skills measured by the ACT Mathematics Test. It is never desirable for test scores or test content to become the sole focus of classroom instruction. However, considered with information from a variety of other sources, the results of standardized tests can help you identify areas of strength and weakness. The activities that follow are examples of sound educational practices and imaginative, integrated learning experiences. As part of a carefully designed instructional program, these activities may result in improved performance on the ACT Mathematics Test—not because they show how to drill students in specific, isolated skills but because they encourage thinking and integrated learning. These activities can help because they encourage the kind of thinking processes and strategies the ACT Mathematics Test requires.

Guiding Principles

- "Students learn more when they are offered opportunities for active application of skills and abilities and time for contemplation." (American Association for Higher Education [AAHE], American College Personnel Association, & Association of Student Personnel Administrators, 1998, p. 5)
- "An effective . . . mathematics program engages students in a variety of learning experiences designed to promote mathematical exploration and reasoning." (Pechman, 1991, p. 35)
- "In order for assessment to support student learning, it must include teachers in all stages of the process and be embedded in curriculum and teaching activities." (Darling-Hammond, 1994, p. 25)

Using Sampling Techniques

College Readiness Standards

- Solve some routine two-step arithmetic problems
- Exhibit knowledge of simple counting techniques
- Compute a probability when the event and/or sample space are not given or obvious
- Exhibit knowledge of conditional and joint probability
- Calculate the average of a list of numbers
- Use Venn diagrams in counting
- Distinguish between mean, median, and mode for a list of numbers

Description of the Instructional Activity

During a class discussion, students could review the concepts of probability and relative frequency, applying the concepts to real-world situations (e.g., rolling a die).

Working in small groups, students could then design and perform an experiment to determine the distribution of colors or varieties of similar objects in a container (e.g., M&M's, marbles, or buttons). After predicting the distribution based on the results of their experiment, the students could look in the container and compare their predicted distribution with the actual distribution.

Each group could report on their experiment, the number of trials, the relative frequencies, and the actual distribution. Students could use that information to plan and perform a series of similar experiments to determine the optimal number of trials needed to accurately predict the distribution.

Depending on the students' knowledge of probability, students could also design experiments based on conditional probability (e.g., taking out two or three objects before replacing them). Students could also use a computer program to model these experiments.

Suggestions for Assessment

Performance Task—Students could be given a different situation (e.g., a container with an unknown number of M&M's or a container of 50 marbles with an unknown number of colors). Using the knowledge they gained from their other experiments, students could design another experiment, including a written rationale. The teacher could use a rubric to evaluate the design and a set of criteria to evaluate the rationale.

Ideas for Progress

- Model and solve real-world problems that involve a combination of rates, proportions, and/or percents
- Construct and analyze Venn diagrams to help determine simple probabilities
- Design and conduct probability investigations (e.g., how the margin of error is determined) and then determine, analyze, and communicate the results

Suggested Strategies/Activities

Students could be introduced to measures of central tendency as different methods of interpreting data for use in decision making. Students could then inquire about how data is organized, displayed, and summarized for reports in industry or politics.

Students could investigate how sampling is used in industry or politics. Topics might include how the margin of error is determined (e.g., poll results that are accurate within 3%) and how political candidates are declared winners by the major networks when a very small percentage of the votes are in. They could explore how different companies or groups in the community choose the number of members to use in a sample from the population. Students could learn about and discuss ways industries and groups use various sampling techniques so that the sample is random and not biased in some way.

Guiding Principles

- "Students . . . should learn mathematics as a process of constructing and interpreting patterns, of discovering strategies for solving problems, and of exploring the beauty and applications of mathematics." (Mathematical Sciences Education Board, Board on Mathematical Sciences, & National Research Council, 1989, p. 66)
- "Learning involves the ability of individuals to monitor their own learning, to understand how knowledge is acquired, to develop strategies for learning based on discerning their capacities and limitations, and to be aware of their own ways of knowing in approaching new bodies of knowledge and disciplinary frameworks." (AAHE et al., 1998, p. 14)
- "Assessment of learning should encompass all aspects of the educational experience." (AAHE et al., 1998, p. 7)

Pascal's Triangle

College Readiness Standards

- Compute a probability when the event and/or sample space are not given or obvious
- Draw conclusions based on number concepts, algebraic properties, and/or relationships between expressions and numbers
- Write expressions that require planning and/or manipulating to accurately model a situation
- Evaluate polynomial functions, expressed in function notation, at integer values
- Manipulate expressions and equations

Description of the Instructional Activity

Students could be given the first few rows of Pascal's triangle and asked to determine the next several lines. Students could provide assistance to a peer by giving hints (e.g., providing a particular entry or suggesting a strategy).

Working in groups, students could identify other sequences that are involved in the triangle (e.g., Fibonacci sequence and triangular numbers). Students could define those sequences algebraically, recursively, or by position in the triangle.

As an algebraic application, students could be asked to expand binomials raised to powers (e.g., $(x + y)^3$) and compare the coefficients with entries in Pascal's triangle. The discussion could be expanded to include the binomial coefficient notation and formula. Also, students could explore binomials in the form $(ax^r + by^s)^t$.

For certain events (e.g., flipping a coin three times), students could list all the combinations, determine the number of occurrences, and compute their probabilities. Students could investigate the connection with Pascal's triangle for determining the numbers of combinations and probability. Students could also explore how $(ax^r + by^s)^t$ is related to flipping coins.

Suggestions for Assessment

Pencil-and-Paper Tasks—Students could be given Pascal's triangle and a series of constructed-response or multiple-choice questions that use entries in the triangle. Students could also explain how Pascal's triangle would aid them or others in expanding binomials or computing probabilities. The teacher could assess the accuracy of the mathematics, the communication of ideas and concepts, and the application of reasoning skills that the students demonstrate.

Linking Instruction and Assessment

Strands: *Numbers: Concepts & Properties; Data Analysis; Expressions, Equations, & Inequalities; Functions*

Ideas for Progress

- Gather, organize, display, and analyze data in a variety of ways to use in problem solving
- Find the probability of simple events, disjoint events, compound events, and independent events in a variety of settings using a variety of counting techniques
- Create expressions that model mathematical situations using combinations of symbols and numbers
- Make generalizations, arrive at conclusions based on conditional statements, and offer solutions for new situations that involve connecting mathematics with other content areas

Suggested Strategies/Activities

Students could conduct a survey to determine whether people (e.g., fellow students, neighbors, local merchants) regularly, occasionally, or never buy lottery tickets, displaying the data in an appropriate format (e.g., circle graph, bar graph). As a group students could then analyze the data, drawing conclusions about people's buying habits with regard to lottery tickets. They could contact a lottery commission and research the way the odds are computed and then use the information obtained to actually calculate the odds of winning. Students could also be asked to investigate and explore situations or conditions that might change the odds of winning. They could investigate the expected value (gain or loss) of someone who buys \$$x$ worth of lottery tickets over a set period of time versus someone who spends \$$y$ over the same period of time.

Guiding Principles

- "Student understanding develops as a result of students' building on prior knowledge via purposeful engagement in problem solving and substantive discussions with other students and teachers in classrooms." (National Center for Improving Student Learning and Achievement in Mathematics and Science, 1997, p. 14)
- "A major purpose of evaluation is to help teachers better understand what students know and make meaningful decisions about teaching and learning activities." (Zemelman, Daniels, & Hyde, 1993, p. 77)
- "As students realize that their performances are being recognized and assessed reasonably, their performances will improve." (Association for Supervision and Curriculum Development [ASCD], 1999, p. 11)

Trigonometric Graphs

College Readiness Standards

- Analyze and draw conclusions based on information from graphs in the coordinate plane
- Write equations and inequalities that require planning, manipulating, and/or solving
- Locate points in the coordinate plane
- Match graphs of basic trigonometric functions with their equations

Description of the Instructional Activity

Students could make a table of common trigonometric values based on their experience with right triangles (e.g., cos 30° or sin 45°). The students could predict the shape of the sine and cosine graphs using those values. Using a graphing calculator, the teacher could then demonstrate how to graph $y = \sin x$ and $y = \cos x$. Using the table or trace feature on the calculator, students could compare their table of values with the graphs and extend their table to include values for larger angles. Using the tables and graphs, students could look for patterns and identify relationships between the two functions (e.g., values for complementary angles or translation of graphs).

Students could explore graphing equations in the form $y = \sin(x + C)$ to obtain an equation that has the same graph as $y = \cos x$. Working in small groups, students could then explore, discover patterns, and make predictions of the effect on the graph when each of the parameters in the equation $y = A \sin(Bx + C) + D$ is changed. They could complete a teacher-produced worksheet designed to help them focus their exploration and to be more methodical, depending on their experience (Caughlan, Myers, & Forrest, n.d.).

Afterward, the class could discuss the effect of parameter changes and the appropriate mathematical terminology. They could practice translating between the different representations: mathematical terminology, graphical, and algebraic.

Suggestions for Assessment

Checklist—The teacher could observe students' learning by asking them leading and focusing questions (e.g., "How can you reflect the graph about the x-axis?" or "How did you reach that conclusion?"). Observed knowledge and understandings could be recorded along with comments regarding partial understandings or misconceptions.

Pencil-and-Paper Tasks—Students could be given a set of equations and asked either to sketch their graphs without using a calculator or to describe, using appropriate mathematical terminology, how each graph would differ from the basic graph, $y = \sin x$ or $y = \cos x$. They could also be given a graph and asked to determine its equation or to describe its features mathematically. These questions could be given in a multiple-choice format.

Ideas for Progress

- Formulate expressions, equations, and inequalities that require planning to accurately model real-world problems (e.g., direct and inverse variation)
- Make generalizations, arrive at conclusions based on conditional statements, and offer solutions for new situations that involve connecting mathematics with other content areas
- Explore geometric models where unit circle trigonometry and basic identities can be used to solve problems

Suggested Strategies/Activities

Some groups could work with the sine function, while others work with the cosine function. A comparison could be made to determine if the various parameters change the graphs in the same ways. The activity could also be extended to include the graphs of the other trigonometric functions. In particular, students could compare and contrast the effect of parameter changes on reciprocal functions.

Students could be given real-world situations that can be modeled using sine waves (e.g., a portion of a roller-coaster track, depth of tidal waters over time, or sound waves). Using data they gather, students could write a modeling equation that approximates the data.

Guiding Principles

- "Students are more likely to understand a scientific or mathematical idea when they consciously try to examine its ramifications and to think about where and when it applies—or does not apply." (Secada, 1997, p. 9)
- "When allowed to explore mathematics with proper instruction and the aid of a computer, students can develop the disposition to question and investigate mathematics more deeply." (ASCD, 1999, p. 21)
- "Observing, questioning, and listening are the primary sources of evidence for assessment that is continual, recursive, and integrated with instruction." (NCTM, 1995, p. 46)

Scaling Factors

College Readiness Standards

- Solve complex arithmetic problems involving percent of increase or decrease and problems requiring integration of several concepts from pre-algebra and/or pre-geometry (e.g., comparing percentages or averages, using several ratios, and finding ratios in geometry settings)
- Solve multistep geometry problems that involve integrating concepts, planning, visualization, and/or making connections with other content areas
- Use relationships involving area, perimeter, and volume of geometric figures to compute another measure
- Use scale factors to determine the magnitude of a size change

Description of the Instructional Activity

Students could review and practice methods for computing the volume and surface area of the following solids: rectangular prism, triangular prism, cylinder, cone, and square pyramid. The teacher could then divide the class into at least five groups. Each group could be assigned one type of solid and provided with about five of those solids, representing a range in size. The students could measure and record in a table the dimensions they need for computing surface area and volume.

If the teacher wants to focus on spatial skills, students could make a 3-D scale model of each solid using a different scale. Students could create a 3-D model by drawing the faces to scale on graph paper (e.g., 1 cm = $\frac{1}{4}$ inch), cutting them out, and taping them together. If the teacher wants to focus on measurement, students could measure each solid using a different unit. Students could record the new data in the table. Then, they could compute and record the surface area and volume using both sets of measurements.

From their data, students could determine the scale factors (how many times larger) for length, surface area, and volume and make some conjectures. They could test these conjectures by measuring some of the solids using another unit of measure. Each group could discuss their findings with the class, and the class could draw conclusions regarding scale factors for different measures of the various solids.

Suggestions for Assessment

Rating Scale—Students could describe in their mathematics journal the technique(s) they used to make their scale models and the conclusions they came to regarding scale factors for the various solids. The teacher could rate students' entries using a scale of 1 to 4 on such criteria as quality of work, accuracy of work, mathematical skills and understandings, organization of information, and communication of ideas.

Anecdotal Notes—Students could be observed while working in groups. The teacher could note whether students are actively participating, correctly calculating the volume and surface area, and making and testing their conjectures.

Ideas for Progress

- Apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures
- Explain, solve, and/or draw conclusions for complex problems using relationships and elementary number concepts
- Examine and compare a variety of methods to find areas of composite figures and construct scale drawings

Suggested Strategies/Activities

Using either 3-D models or computer software (e.g., *Geometric Supposer*), students could explore the effect changing only one or two of its dimensions has on the volume and surface area of a solid. They could also identify those faces that are similar to the original solid and describe the distortion of those faces that are not similar.

Linking Instruction and Assessment

Strands: *Data Analysis; Expressions, Equations, & Inequalities; Graphical Representations; Functions*

Guiding Principles

- "Rich learning experiences . . . enable students to make connections through curricula integrating ideas and themes within and across fields of knowledge and (through) establishing coherence among learning experiences within and beyond the classroom." (AAHE et al., 1998, p. 4)
- "[Teachers] orchestrate discourse by deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty." (NCTM, 1991, p. 35)
- "Learning requires frequent feedback if it is to be sustained, practice if it is to be nourished, and opportunities to use what has been learned." (AAHE et al., 1998, p. 10)

Linear Programming

College Readiness Standards

- Analyze and draw conclusions based on information from graphs in the coordinate plane
- Find solutions to systems of linear equations
- Solve problems integrating multiple algebraic and/or geometric concepts
- Evaluate polynomial functions, expressed in function notation, at integer values

Description of the Instructional Activity

The teacher could guide the students through the process by providing a real-world situation to optimize (e.g., profit for a company that produces two items with different profit and production rates, using a fixed number of employees, machines, and work hours). Using the parameters given, the students would write, with the teacher's guidance, a function to model the profit and translate the constraints into a system of inequalities.

The students could then graph the feasible region by graphing the system of inequalities. To find an optimal solution, students could evaluate the profit function for each of the vertices of the feasible region, for three points within the region, and for three points outside the region.

The class could discuss which point represented the greatest profit. The teacher could then demonstrate several other situations and have the students brainstorm where the point of optimization lies. The linear programming theorem could be discussed.

Suggestions for Assessment

Performance Task—Students, either individually or in small groups, could be given a situation similar to those modeled in class in which they need to determine decision variables, an optimization function, a system of inequalities for the constraints, the feasible region, and an optimal solution. Or students could be given different constraints for a situation modeled in class and asked to describe the effect of each constraint on the feasible region and the optimal solution. The teacher could use a rubric to score students' work on such criteria as quality of work, accuracy of work, mathematical skills and understandings, organization of information, and communication of ideas.

Ideas for Progress

- Gather, organize, display, and analyze data in a variety of ways to use in problem solving
- Create and use basic families of functions (which include linear, absolute value, and quadratic) to model and solve problems in common settings
- Represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)

Suggested Strategies/Activities

Students could research and describe contributions to linear programming by people such as George Dantzig, Narendra Karmarkar, and L. G. Khachian.

Students could contact local agencies or businesses in their community to investigate how they employ linear programming or optimization procedures. Business applications might include some type of scheduling (e.g., airline, train, truck) and routing of information over communication networks. In some instances of linear programming, it is not an efficient use of time to test every corner point for cases that have multiple corner points. Students could be introduced to simplex and ellipsoid methods to find the corner point that represents maximum profit, for instance. Other ideas and lesson plans are available on the Internet. See, for example, The Gateway to Educational Materials (GEM) at http://www.thegateway.org.

PUTTING THE PIECES TOGETHER

ACT developed this guide to show the link between the ACT Mathematics Test results and daily classroom work. The guide serves as a resource for teachers, curriculum coordinators, and counselors by explaining what the College Readiness Standards say about students' academic progress.

The guide explains how the test questions on the ACT Mathematics Test are related to the College Readiness Standards and describes what kinds of reasoning skills are measured. The sample instructional activities and classroom assessments suggest some approaches to take to help students develop and apply their reasoning skills.

WHERE DO WE GO FROM HERE?

ACT recognizes that teachers are the essential link between instruction and assessment. We are committed to providing you with assistance as you continue your efforts to provide quality instruction.

ACT is always looking for ways to improve its services. We welcome your comments and questions. Please send them to:

College Readiness Standards Information Services
Elementary and Secondary School Programs (32)
ACT
P.O. Box 168
Iowa City, IA 52243-0168

"A mind, stretched to a new idea, never goes back to its original dimensions."

— Oliver Wendell Holmes

WHAT OTHER ACT PRODUCTS AND SERVICES ARE AVAILABLE?

In addition to the College Readiness Standards Information Services, ACT offers many products and services that support school counselors, students and their parents, and others. Here are some of these additional resources:

ACT's Website—**www.act.org** contains a host of information and resources for parents, teachers, and others. Students can visit **www.actstudent.org**, which is designed to aid students as they prepare for their next level of learning.

PLAN—a comprehensive assessment program designed to improve tenth-grade students' postsecondary planning and preparation and to enable schools to assist students and their parents in this important process.

EXPLORE—an eighth- and ninth-grade assessment program designed to stimulate career explorations and facilitate high school planning.

WorkKeys®—a system linking workplace skill areas to instructional support and specific requirements of occupations.

ACT Online Prep™—an online test preparation program that provides students with real ACT tests and an interactive learning experience.

The Real ACT Prep Guide—the official print guide to the ACT, containing three practice ACTs.

DISCOVER®—a computer-based career planning system that helps users assess their interests, abilities, experiences, and values, and provides instant results for use in investigating educational and occupational options.

BIBLIOGRAPHY

This bibliography is divided into three sections. The first section lists the sources used in describing the ACT Program, the College Readiness Standards for the ACT Mathematics Test, and ACT's philosophy regarding educational testing. The second section, which lists the sources used to develop the instructional activities and assessments, provides suggestions for further reading in the areas of thinking and reasoning, learning theory, and best practice. The third section provides a list of resources suggested by classroom teachers.

(Please note that in 1996 the corporate name "The American College Testing Program" was changed to "ACT.")

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3. Resources Suggested by Classroom Teachers

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bigchalk The Education Network.
http://www.bigchalk.com

Cloudnet. Multicultural Math Activities.
http://www.cloudnet.com/~edrbsass/edmulticult.htm#math

Educational REALMS: Resources for Engaging Active Learners in Math and Science.
http://www.stemworks.org/realmshomepage.html

The Educator's Reference Desk.
http://www.eduref.org

The Gateway to Educational Materials.
http://www.thegateway.org

Johns Hopkins University & Howard University Center for Research on the Education of Students Placed at Risk (CRESPAR).
http://www.csos.jhu.edu/crespar/crespar.html

Learning Network.
http://www.teachervision.com

The Math Forum. Math Forum @ Drexel.
http://mathforum.org

Mathematical Association of America (MAA). MAA Bookstore.
http://www.maa.org/ecomtpro/timssnet/common/tnt_frontpage.cfm

Minority Student Achievement (MSA) Network.
http://www.eths.k12.il.us/MSA/msanetwork.html

National Council of Teachers of Mathematics. Illuminations: Principles & Standards for School Mathematics.
http://illuminations.nctm.org/standards.aspx

Northwest Regional Educational Laboratory. Library in the Sky.
http://www.nwrel.org/sky/

Northwest Regional Educational Laboratory. Mathematics and Science Education Center.
http://www.nwrel.org/msec/

PBS TeacherSource – Math.
http://www.pbs.org/teachersource/math.htm

Texas Instruments. EXPLORATIONS™ Curricular Materials.
http://education.ti.com/us/activity/books/overview.html

U.S. Department of Education. Education Resource Organizations Directory.
http://bcol02.ed.gov/Programs/EROD/org_list.cfm?category_ID=SEA